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An investigation into cognitive decline and its costs in retirement

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ABSTRACT
Cognitive decline is a reduction in an individual's cognitive abilities. The three factors that this paper discusses that result in such a reduction are: age-related effects, the effect of not using one's mental capabilities and the effect of retiring. The effect of retiring may be viewed as a combination of the first two effects. After discussing the factors affecting cognition, a Markov chain is constructed with three states which are: normal cognition, demented cognition and the dead state. Using the model, males are shown to have a lower expected lifetime in the demented state compared to females. Whilst in the demented state, a person is expected to incur costs of care due to poor cognition. It is found that a male aged 60 is expected to incur R3 220 000 in costs of care whilst a female aged 60 is expected to incur R4 940 000. Females have a higher expected cost of care because they are expected to live longer in the demented state than males. For individuals who have not saved sufficiently for retirement, these costs would exacerbate their lack of resources.

KEYWORDS
Cognitive decline; retirement; dementia

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1. INTRODUCTION

1.1 According to Zweifel, Felder & Meiers (1999), the world has experienced a trend of an ageing population in developed countries over the past 30 years. The number of people living past the age of 80 has increased and this trend is expected to continue into the near future (Zweifel, Felder & Meiers, 1999). It has been shown that the degree of cognitive decline is positively correlated with an increase in age, but very little has been done to prevent cognitive decline with age (Yankner, 2000).

1.2 From an individual's perspective, the effects of cognitive decline can range from simple forgetfulness to late dementia in which case the brain can no longer tell the body how to function (Reisberg, 1984). Furthermore, Reisberg (1984) suggests that these effects may necessitate professional care for the individual which may be costly.

1.3 It is of interest to know how retirement affects the cognition of a person and how a reduced cognition affects an individual's medical costs in retirement. Knowledge of this allows for full financial planning to be done for retirement, especially for those who wish to retire early due to cognitive decline. The longer a person lives in retirement, the higher the total

wealth required for a given amount of consumption in retirement (Cocco & Gomes, 2012). This leads to the question of how long do people who experience cognitive decline live in retirement as this affects the total cost of care that they might incur.

1.4 A key aspect, as suggested above, is the medical costs for a person who experiences cognitive decline. According to Reisberg (1984), a person would need professional care when one can no longer recall vital information about oneself such as one's name, address or the current year. A form of cognitive decline is dementia and medical costs do arise due to being in this state of reduced cognition (Wolstenholme et al., 2002). Estimating the total medical expenses, which takes into account an individual's level of cognition, will be valuable because it will aid in determining the effect of cognitive decline on an individual's retirement adequacy requirements.

1.5 The age at which a person retires may affect the degree of cognitive decline that the person experiences. Cognitive decline would also accelerate in retirement due to a lack of mental exercise that would have otherwise been present at work; this is known as the "mental retirement" effect. (Rohwedder & Willis, 2010)

1.6 This paper will explore cognitive decline and its associated costs in retirement. The aims of this paper and its actuarial relevance is highlighted in section 2. Section 3 discusses the factors which affect cognition. Section 5 describes a model of dementia, normal mental functioning and death, and the data used in this model is described in section 4. The results and limitations of the model are discussed in section 6 and section 7 states the conclusion of the paper.

2. AIMS

2.1 The three major aims of this paper are to:
— discuss the factors that influence the onset of cognitive decline;
— investigate how dementia, which is a form of cognitive decline, affects one's expected future lifetime compared to an individual in normal cognitive health; and
— estimate the expected costs that are incurred in retirement due to experiencing dementia.

2.2 There are three factors which will be discussed that influence cognitive decline. This includes retirement effects on cognitive decline, age-related effects and the "use it or lose it" hypothesis. The medical expenses associated with dementia are then considered. A model is built to estimate the expected present value (EPV) of these costs in retirement. Note, the discussion which relates to first aim outlined in 2.1 is for cognitive decline in general whilst the next two aims relate to dementia.

2.3 The discussion of the factors that influence cognitive decline will provide an increased awareness of it and may allow individuals to make better decisions regarding their cognition. The discussion will also provide a background to the model that will be constructed.

2.4 The estimation of expected costs in retirement of dementia will be useful to insurers who cover dementia-related disorders[1,2]. This research will provide an estimate of the EPV of the costs of care in retirement which may aid in determining the appropriate amount of premium to charge in providing the benefit. Notably, the insurers listed in footnotes 1 and 2 protect against Alzheimer's disease which is the most common form of dementia[3], but it is not the only form.

2.5 Since all types of dementia-related risks are not covered by the risk products offered by Momentum and OUTsurance (see footnotes 1 and 2), individuals are expected to deal with the risks that are not covered by insurers. Thus, this research will enable better financial planning for individuals who experience some form of dementia as it estimates the EPV of medical costs relating to dementia.

2.6 This research can be viewed as a preliminary investigation into cognitive decline and the associated costs in retirement. This could be used to aid future research into cognitive decline in retirement in South Africa.

3. COGNITIVE DECLINE

3.1 Age-related effects

3.1.1 In discussing the age-related effects on cognition, it is important to define cognition. It is proposed to separate cognitive abilities into two parts – "fluid intelligence" and "crystallised intelligence". Fluid intelligence is one's ability to reason using concepts and to solve problems using unfamiliar information or procedures. Crystallised intelligence consists of habits and knowledge that one acquires through education and experience. (McArdle et al., 2002)

3.1.2 When Cattell (1943) first proposed the above two constructs of intelligence, it was suggested that fluid intelligence decreases with age for adults while crystallized intelligence increases with age. This is, to some extent, supported by McArdle et al. (2002). McArdle et al. (2002) initially shows an increasing trend for both fluid and crystallized intelligence at early ages; however, a decreasing trend for fluid intelligence emerges around the age of 20 while crystallized intelligence starts to decrease around the age of 40.

3.1.3 A study by Salthouse (2009), which measured intelligence using a different model of intelligence, came to a similar conclusion – age-related decline in one's abilities starts around the age of 20. The speed at which one can process new information is a significant predictor of age-related cognitive decline (with higher speeds indicating a lower risk of cognitive decline) and the decline in processing speed is expected to start around the age of 30 (Deary et al., 2009).

3.1.4 Altogether, the literature indicates that cognitive decline occurs much earlier than retirement. This leads to an important question – if a person lives long enough would the individual become demented? According to Yankner (2000), if the answer to this question is yes and mental decline cannot be completely prevented then advances in medical care which extend an individual's life to higher ages are a hollow victory. This paper will not elaborate

1 Momentum. The cost of surviving illness – your biggest lifetime expense? https://www.momentum.co.za/for/you/products/life/myriad/critical-illness, accessed 30 August 2017

2 OUTsurance. Critical illness table. https://www.outsurance.co.za/life-insurance/critical-illness-cover-table/, accessed 30 August 2017

3 Alzheimer's Society. What is dementia? https://www.alzheimers.org.uk/info/20007/types_of_dementia/1/what_is_dementia, accessed 30 August 2017

further on this issue of the effect of longevity on cognition, but will consider the costs that one will face in these higher ages due to dementia.

3.2 The "use it or lose it" hypothesis

3.2.1 Hultsch et al. (1999) proposed the "use it or lose it" hypothesis which states: when an individual fails to engage in intellectually stimulating activities then he or she will experience cognitive decline. The decline experienced will be over and above that experienced by "normal" age-related cognitive decline which is discussed in section 3.1 (Hultsch et al., 1999).

3.2.2 These mentally stimulating activities serve to slow the rate of cognitive decline experienced by an individual (Hultsch et al., 1999). These activities could be part of daily life or they could form part of an individual's work-related activities (Rohwedder & Willis, 2010). Collectively, these activities form part of "mental exercise" which was suggested by Rohwedder and Willis (2010) as a way to maintain one's cognitive ability or slow the rate of cognitive decline.

3.2.3 Hultsch et al. (1999) found a statistically significant relationship between changes in mentally stimulating activities performed and changes in the cognitive functioning of an individual's brain which suggested that intellectually stimulating activities serve to delay the onset of cognitive decline. This is weakly supported by Salthouse (2006) as there was great difficulty in finding an appropriate sample to test the use it or lose it hypothesis. Rohwedder & Willis (2010) found similar results to Hultsch et al. (1999); however, these results which suggested that mental exercise could delay cognitive decline only related to retired individuals who no longer engage in workplace activities.

3.2.4 Gatz (2005) discusses the idea of a "mental reserve" which is the ability of an individual to compensate for the deterioration of the brain which has already occurred. It is suggested that an individual can perform certain exercises to increase their mental reserve. This increased mental reserve will slow the rate of cognitive decline although the evidence to support the mental reserve has been weak. (Gatz, 2005)

3.2.5 Gatz (2005) also suggests that the activities which could serve as mental exercise could be educational leisure activities; however, activities that increase memory – such as mnemonic strategies for memory improvements – are likely to have a stronger effect. Another strategy which is proposed by Penedo & Dahn (2005) is physical exercise. It is suggested that physical exercise does not only provide physical health benefits, but can also serve to delay normal age-related cognitive decline (Penedo & Dahn, 2005).

3.2.6 Ball et al. (2002) conducted a controlled trial in which they tested whether three "cognitive training" exercises could help in improving the mental health of pensioners who lived independently. The results from this trial showed that memory training, reasoning training and speed of processing training were found to be effective in improving the cognitive functioning of retired independent-living adults (Ball et al., 2002). These results are similar to the exercising of the two parts of cognition – fluid and crystallised – as suggested by McArdle et al. (2002).

3.3 The effects of retirement

3.3.1 There are two views pertaining to the effect of retirement on an individual's cognition. The first hypothesis is that retirement is beneficial as it relieves people of the burden of work which may have been the cause of stress. This would potentially have a good effect on the mental health of individuals. (Mazzona & Peracchi, 2017)

3.3.2 Alternatively, Rohwedder & Willis (2010) proposed that retirement is harmful to the cognitive abilities of an individual. It is suggested that due to a decrease in an individual's engagement in cognitively intensive tasks at work, the individual would experience a decrease

in cognitive abilities (Rohwedder & Willis, 2010). This reflects the use it or lose it hypothesis described in section 3.2.

3.3.3 To test the above two hypotheses, regression techniques can be used (Coe et al., 2012); however, there are some important aspects that first need to be considered. The first is reverse causation – people select into retirement due to poor cognition (Mazzona & Peracchi, 2017). Secondly, it is likely that retirement does not have an immediate (contemporaneous) effect on cognition, rather cognition decreases slowly after retirement (Coe et al., 2012). The third issue pertains to heterogeneity across occupation groups (Mazzon & Peracchi, 2017). The first issue can be dealt with by using the instrument variable technique. The instrument variable technique is an econometric technique of determining casual direction (Newhouse & McClellan, 1998). For the second issue, the time in retirement can be used in the model as an additional predictor.

3.3.4 The third issue is resolved by constructing an index with a range of 1–10 which measures the level of physical effort that is required by a particular job. The data pertaining to people's cognitive abilities is then split between low values (1–5) and high values (6–10) of the index. Using this method, Mazzona & Peracchi (2017) found that retirement was a significant predictor of improved cognition for people with physically strenuous jobs. For those with a low index value, retirement was shown to have a negative effect on cognition; however, this effect was not significant. (Mazzona & Peracchi, 2017).

3.3.5 Four papers were found which used the instrument variable technique and used time in retirement as a predictor of an individual's level of cognition. Three of these four papers are: Mazzonna & Perrachi (2017), Rohwedder & Willis (2010) and Bonsang, Adam & Perelman (2012) and these papers showed a negative causal relationship between the amount of time in retirement and the level of cognition. Coe et al. (2012), the fourth paper, showed no such causal relationship between the amount of time in retirement and the level of cognition.

3.3.6 The retirement effect, however, may just be a combination of the previous two effects discussed in sections 3.1 and 3.2. Stine-Morrow (2007) argues that the choice of how effort is allocated to cognitive tasks in retirement is a significant predictor of cognition. This reflects the use it or lose it hypothesis by Hultsch et al. (1999). The other effect proposed by Stine-Morrow (2007) is senescence or the age-related effect discussed by Salthouse (2009).

3.3.7 The main argument by Stine-Morrow (2007) is that older people who start to doubt their ability to engage in intellectually stimulating activities due to their age – since older people are stereotyped to be less intellectually capable – may not even engage in these activities at all. By the use it or lose it hypothesis (Hultsch et al., 1999), this causes older people to slowly lose their ability to engage in intellectually stimulating tasks. The reduction in their mental abilities further makes older people doubt their abilities. This is a vicious cycle which can only be ended by attempting to engage in intellectual activities. Essentially, losses in cognition come for free (no effort is required) while gains or preservation requires intentional effort on the part of individual (Stine-Morrow, 2007).

3.4 Model motivation and outline

3.4.1 As discussed above, there are various factors which may influence a person's cognition in retirement and the discussion in sections 3.1 to 3.3 highlights the salient factors which affect cognition. The next step is to model the impact of cognitive decline on an individual's life. Due to limited data, the model will focus on a form of cognitive decline, namely dementia. Dementia is a form of cognitive decline where a person has trouble with concentration, memory or even difficulty having a conversation[4].

4 Alzheimer's Society. Symptoms of dementia. https://www.alzheimers.org.uk/info/20007/types_of_dementia/1/what_is_dementia/2, accessed 23 May 2017

3.4.2 The aim of the model is to provide a quantification of the impact of dementia on an individual to enable a better understanding of the financial consequences of dementia. Specifically, it aims to model the transitions between a normal state of cognition, a demented state of cognition and the dead state which will allow for the calculation of transition probabilities. Using these probabilities, the EPV of the costs associated with dementia – such as costs of care which is needed due to cognitive problems (Wolstenholme et al., 2002) – are estimated. The transition probabilities are also used to estimate the expected future lifetime for an individual in the demented state.

3.4.3 Section 4, below, outlines the data that is required for the model and the sources that are used to obtain this data. Section 5 constructs the framework of the model and section 6 will discuss the results of the model together with its limitations.

4. DATA

4.1 Mortality

4.1.1 Mortality data for the South African retired population was acquired from a report on pensioner mortality conducted by the Continuous Statistical Investigations (CSI) Committee of the Actuarial Society of South Africa[5]. This investigation was conducted over a six-year period from the beginning of 2005 to the end of 2010. This data was provided to the CSI Committee by 22 large pension funds in South Africa including the Government Employees Pension Fund (GEPF) which dominated the experience of the study. Membership to the GEPF is compulsory for government employees in South Africa[6]. The private sector might experience a different rate of mortality; however, the study provides results in the form of a combined set of data which does not differentiate mortality between the public sector and the private sector. The results of this paper should therefore be used with this in mind.

4.1.2 The resulting dataset has values for q_x and $\mu_{x+0.5}$ for the ages 50–110 inclusive for both males and females. The definition of q_x is the probability of a life aged x dying before he or she reaches the age of x+1 and $\mu_{x+0.5}$ is the force of mortality at the midpoint between ages x and x+1 (Appendix A).

4.2 Mortality adjustment

A 20-year longitudinal study was done to investigate the effect of cognitive decline on mortality. This study uses details of 921 people who were from Great Britain. The Abbreviated Mental Test (AMT) was used to determine a person's cognitive functioning. Subjects were categorised into three levels: severe impairment, slight impairment and normal mental functioning. A Cox proportional hazards model was then used to determine the effect of the level of cognition on mortality. People in the normal mental functioning category (normal people) were the base and have a hazard ratio of one. The results showed that people with severe impairment had a hazard ratio of 3.3 compared to normal people and slightly impaired people had a hazard ratio of 1.7 compared to normal people. A weighted average of these two hazard ratios (where the weights are the number of people in each category) is 2.029. This means that people who have some form of cognitive impairment, on average, have a 2.029 times higher force of mortality compared to normal people. (Gale, Martyn & Cooper, 1996)

5 Actuarial Society of South Africa. Pensioner Mortality Investigation, 2005–2010. http://www.actuarialsociety.org.za/Societyactivities/CommitteeActivities/ContinuousStatisticalInvestigation(CSI).aspx, accessed 22 May 2017

6 GEPF. Who is a GEPF pensioner? http://www.gepf.gov.za/index.php/pensioners/article/who-is-a-gepf-pensioner, accessed 7 September 2017

4.3 Incidence rates for dementia

4.3.1 Ott et al. (1998) performed a prospective study on the incidence rates of dementia and it was found that the incidence rate averaged 1.07% for people above the age of 55. Ott et al. (1998) also calculated the incidence rates for people in five year age-bands and by sex. However, Ott et al. (1998) does not have sufficient observations at the later ages and the incidence drops around age 90. An incidence rate of dementia which increases by age is supported by a review paper by Fratiglioni et al. (2000); the paper also shows that an increasing incidence rate of dementia is consistent across different countries. This is to be expected because a person's cognition decreases by age (Salthouse, 2009; Deary et al., 2009). Therefore, the data from Ott et al. (1998) was supplemented with data from a study specifically for people aged 90+ (Corrada et al., 2010).

4.3.2 Despite using two data sets (see Appendix B), there are still problems with the data. Firstly, there is a drop in male incidence rates from the age-band 60–64 to the age-band 65–69. This is inexplicable and is inconsistent with the discussion above. Secondly, there is a massive jump in male incidence rates in the age-band 90–94 from Ott et al. (1998) to data from Corrada et al. (2010) for the same age-band. The corresponding jump in incidence rates for females is smaller than that of males, but it is still large. Whether such jumps in incidence rates are to be expected is currently unknown.

4.4 Medical costs

4.4.1 A longitudinal case study done in Great Britain which included 100 participants estimated the medical costs – which included the cost of care, if needed – per individual based on their level of cognition. The study showed that costs of care increased with an increase in severity of cognitive decline. This increase in severity causes a transfer from home-based care to institutional care which results in an increase in costs. Home-based care is care provided to an individual at their home by a dedicated carer. Institutional care is provided in a separate facility from an individual's home and often provides 24-hour supervision, skilled nursing care and psychological services (Gellman & Turner, 2013). The transfer from home-based care to institutional care represents a significant increase in costs for an individual and institutional care is estimated to be 69% of the total cost of caring for dementia patients. (Wolstenholme et al., 2002)

4.4.2 An increasing value for medical costs is justified as a person loses their ability to deal with the activities of daily living, their need for care increases (Moore, Zhu & Clipp, 2001). The increase in costs will be represented by a move from home care to institutional care in the form of 24-hour nursing care which is one of the recommended care institutions for dementia[7]. The estimate of South African home-based care in 2017 ZAR is approximately R16 000 per month, or R192 000 per year, and the estimate for 24-hour nursing care in South Africa is around R25 000 per month, or R300 000 per year[8].

4.5 Institutionalisation

A study was conducted into the reasons for placing dementia patients into institutional care and the time it takes for the transfer from home-based care to institutional care to occur. In this study, it was found that the mean duration spent in home-based care before institutionalisation was 4.458 years for males and 2.5 years for females. (Thomas et al., 2004)

7 A Place for Mom. Dementia Care Guide. http://www.aplaceformom.com/dementia-care, accessed 30 August 2017

8 IOL Personal Finance. Frail Care: what you should know. https://www.iol.co.za/personal-finance/frail-care-what-you-should-know-1860431, accessed 30 August 2017

4.6 Medical Inflation

4.6.1 Medical inflation is often found to be above the general inflation in many countries (Charlesworth, 2014) and this is the case in South Africa (Still, 2016). Therefore, medical inflation, and not simply Consumer Price Index (CPI), needs to be examined and considered when working with medical costs in the future. The excess of medical inflation over the CPI can be explained in three parts which are:

—an increased demand for services (which is caused by the population becoming older and sicker or if there is adverse selection in a medical scheme population or a combination of both);

—supply side factors (which are caused by suppliers' costs increasing due to new technologies or pharmaceuticals); and

—tariff increases exceeding CPI (Still, 2016).

4.6.2 Medical inflation data (Appendix C) from Stats SA is in the form of a year-on-year medical inflation figure for each month from January 2012 to April 2016 (Still, 2016: 80). This will be averaged and then used as the expected medical inflation for future medical costs. A further breakdown of the medical inflation figures into the components of medical products, doctors, dentists, hospital services and medical insurance can be obtained from the additional tables for CPI publications[9]. The components with the highest weightings are: medical products, doctors and medical insurance.

4.7 Discount rate

The model discussed below will discount the costs incurred by a pensioner. Butler & Van Zyl (2012) stated that some households would prefer to deal with their longevity risks in retirement by purchasing a commercial retirement annuity with their wealth. The average return on a retirement annuity by a large South African insurer is 9.7%[10] per year. This is considered appropriate as this is a return from a product that a pensioner might use. Therefore, this is the return that is used in the model.

5. MODEL

5.1 The aim of this model is to estimate the total medical costs for a person in dementia. As a first step, a continuous time Markov process is constructed. The use of a Markov process assumes that the Markov property is appropriate. Commenges & Joly (2004) developed a multi-state model for dementia, institutionalisation and death in which the authors did not drop the Markov assumption; the reason being is to keep computations simple. Similarly, this study will also keep the Markov assumption to retain the relative simplicity of calculations.

5.2 The state space of the Markov process contains three states: normal (n), demented (dem) and dead (d). Symbolically, the state space can be written as $S = \{n, d, dem\}$. The dead state is an absorbing state and it is assumed that once a person enters the demented state, he or she shall not return to the normal state. The transition intensities from one state to another are illustrated in Figure 1. Note, the transition intensities are age dependent; this means that the transition intensities are assumed to be constant in the interval $(x, x+1]$ where x is an exact age.

9 Stats SA – Prior Publications (Archive). http://www.statssa.gov.za/?page_id=1866&PPN=P0141&SCH=6763, accessed 30 August 2017

10 Old Mutual. Annual Investment Update – Platinum Pension. https://www.oldmutual.co.za/docs/default-source/corporate/products-services/employee-benefits/retirement-investments/annuity-portfolios/platinum-pension/platinumpensioninvupdate.pdf?sfvrsn=6, accessed 29 May 2017

5.3 The transition intensity from normal to dead is based on data from the CSI investigation that was discussed in section 4.1. The transition intensity from normal to demented is based on data discussed in section 4.3. This transition intensity is allowed to vary by age, as the incidence rate of dementia increases with increasing age (Fratiglioni et al., 2000). Finally, people who experience dementia (a form of cognitive decline) are expected to have a higher mortality than those in normal health (as discussed in section 4.2). Therefore, the transition intensity from the demented state to the dead state is assumed to be 2.029 times higher than the transition intensity from the normal state to dead state for all ages (this is also described in section 4.2).

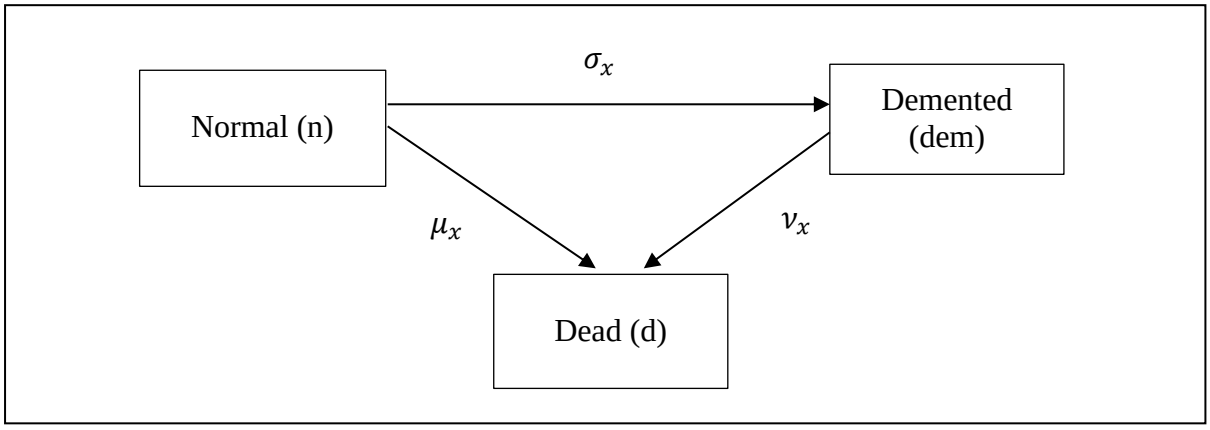


FIGURE 1. The continuous time Markov chain: transition intensities

5.4 Using Kolmogorov's forward equations (Ross, 2014: 389), the transition probability formulae were derived (Appendix D). These formulae were then used to determine the respective transition probabilities using the data described in section 4. An example of a transition probability is ${}_tP_x^{(n,dem)}$; this is the probability that an individual in the normal state, who is currently aged x exactly, will be in the demented state at age x+t. When t=1, the subscript in the transition probability denoting t will be omitted for the sake of simplicity. Using these probabilities, Dickson, Hardy & Waters (2013) showed that the complete expected future lifetime in state $j \in \{n, dem\}$ can be calculated by equation (1). The expected future lifetime will be estimated for individuals either in the normal or demented state.

$$\overset{o}{e}_x = \int_0^\infty {}_tP_x^{(j,j)}dt \qquad (1)$$

5.5 The Markov process represents the stochastic part of the model. A deterministic component is now described for the demented state. In the demented state, it is assumed that there are two forms of care that a patient can receive: home-based care or institutional care. These two forms of care represent two possible sub-states within the demented state. It is assumed that for a person who enters the demented state, he or she will first start receiving home-based care. After a given period of time, if the individual is still alive, he or she would move into institutional care. For males, it is assumed that it takes four years to move from home-based care to institutional care and for females it is assumed that it will take three years (these values are based on the discussion in 4.5).

5.6 While an individual is alive in the demented state, the individual will incur costs for the care that he or she will receive and these costs are assumed to occur at the end of each month. It is further assumed that a person only starts incurring costs in the year after he or she has entered the demented state. There will be two levels of costs corresponding to the two different

levels of care mentioned in section 5.5 which are home-based care and institutional care. These costs would only be incurred as long as the individual remains alive and in the demented state – this is, essentially, a monthly annuity payable for life in arrears. As mentioned in section 4.4, these costs are R16 000 per month for home-based care and R25 000 per month for institutional care in the South African market. The costs are also assumed to be the same for both males and females. The step-up in cost of care would only occur when the individual moves from home-based care to institutional care, which is assumed to be four years for males and three years for females.

5.7 For the modelling exercise, these costs will be converted to yearly figures of R192 000 and R300 000 per year for home-based care and institutional care respectively (note there is an increase of R108 000 in costs from home-based to institutional care). These costs are in 2017 ZAR and will be referred to as the real costs of care. Since the costs that are incurred by the demented individual are medically related, the model allows for the nominal amounts of these payments to be inflated in line with medical inflation. The most heavily weighted components of medical inflation are medical products, doctors and medical insurance, according to the discussion in 4.6. Therefore, the nominal costs of home-based care and institutional care are assumed to increase at a constant rate of 8.71% per year, which is the average of the year-on-year medical inflation figures which can be found in Appendix C.

5.8 The medically inflated costs will then be discounted using a flat, nominal discount rate. This rate will be 9.7% per year as discussed in section 4.7. The effect of the medical inflation and the discount rate can be summarised into a single interest rate, j, which would be a real interest rate taking into account both the nominal interest rate and medical inflation. Therefore, the discount factor $V=(1+j)^{-1}$ will then be used to discount all real payments occurring in the demented state.

5.9 The annuity factor, $a^{(12)}_{60+k}$, is the EPV of an annuity to an individual starting at age 60+k with k being the variable of summation which will be used below. Age 60 is chosen as it is the official retirement age in South Africa for both males and females[11,12]. The annuity is paid monthly in arrears while the individual is in the demented stated – or in other words paid until a person who is in dementia dies since a person in dementia can only move into the dead state. According to Spurgeon (2011: 129), the monthly whole life level annuity for a person aged exactly x, who is in the demented state, can be approximated by equation (2). The formula for a_x as per Dickson, Hardy & Waters (2013: 112) in the current context is shown in equation (3).

$$a^{(12)}_x \cong a_x + \frac{11}{24} \tag{2}$$

$$a_x = \sum_{k=1}^{110-x} V^k {}_kp^{(dem,\,dem)}_x \tag{3}$$

11 Trading economics. Retirement age – men. https://tradingeconomics.com/south-africa/retirement-age-men, accessed 18 August 2017

12 Trading economics. Retirement age – women. https://tradingeconomics.com/south-africa/retirement-age-women, accessed 18 August 2017

5.10 Equation (4) gives the EPV of costs given that a person enters the demented state between the period (60+k−1, 60+k]. It will discount the costs to age 60+k using all the assumptions mentioned above. The equation, for males, is shown below.

$$\text{EPV(Cost of care | Demented by age 60+k)}=192000a_{60+k}^{(12)}+108000V^{4}{}_{4}p_{60+k}^{(\text{dem, dem})}a_{60+k+4}^{(12)} \quad (4)$$

5.11 The equation above can be interpreted as an annuity of R192 000 per year, payable monthly in arrears, plus an annuity of R108 000 per year (the increase from R192 000 per year to R300 000 per year), payable monthly in arrears if the male survives long enough to be institutionalised (four years later according to the assumptions). This would result in payments of R192 000 per year, payable monthly, for the first four years, and R300 000 per year, payable monthly, thereafter. Since the mortality data runs up to the age of 110, the costs will not increase from R192 000 per year if a male enters the demented state for the first time at the age of 106 or higher. This is because four years will not have elapsed in order for the individual to be institutionalised. A similar case would be for females, where the age would be 107 due to the three year time to institutionalisation.

5.12 The EPV (Cost of care) will be determined over the remaining lifetime of the individual (note: this is no longer a conditional calculation). The EPV will be done with the starting age of 60 as this is the official retirement age in South Africa (as mentioned in 5.10). The calculation will be done separately for males and females. This value is expected to be lower than the values obtained from equation (4) as it incorporates the probability of remaining in the normal state and the probability of transitioning into the demented state.

$$\text{EPV (Cost of care)}=\sum_{k=1}^{50}{}_{k-1}P_{60}^{(n,n)}P_{60+k-1}^{(n,\text{dem})}V^{k}\text{EPV(Cost of care | Demented by age 60+k)} \quad (5)$$

6. RESULTS AND ANALYSIS

6.1 Results

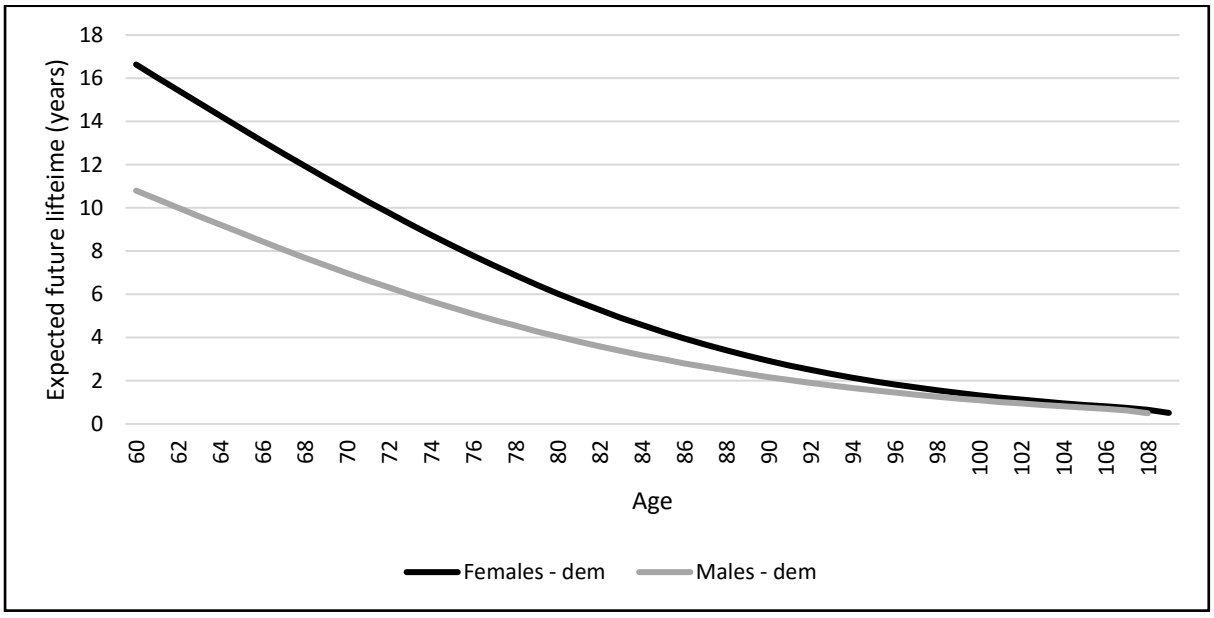
6.1.1 Graph 1 shows the expected future lifetime of individuals who enter into the demented state at varying ages. Females have a higher expected future lifetime at all ages, but this difference decreases in the later ages. This is expected as women live longer than on average (Austad, 2006). Graph 2 shows the expected future lifetime in the normal and demented state for males. Around the age of 60, the expected future lifetime in the normal state is higher than that of the demented state. This is due to the total transition intensity out of the normal state (normal to dead + normal to demented) is lower than the transition intensity from the demented state to the dead state (which is 2.029 times normal to dead). As the transition intensity from normal to demented increases in the later ages, the total transition intensity out of the normal state increases relative to the transition intensity from demented to dead. This is particularly true for the normal to demented transition intensity. Hence, the expected future lifetime in the normal state decreases relative to that of the demented state. The same is applicable for females and the graph is in Appendix E.

6.1.2 Graph 2 shows the results for equation (4). This is the EPV of the cost of care in the demented state given that the transition to the demented state occurs at a specific age. Assuming a person enters the demented state at age 60, the EPV of future costs for a female would be approximately R4 940 000 while for a male the approximate EPV of costs would be R3 220 000. Both of these costs are significant and can have a material effect on an individual's

finances in retirement. Given that only 10% of South African's save enough for an acceptable retirement[13], these costs would exacerbate the individual's lack of resources in retirement.

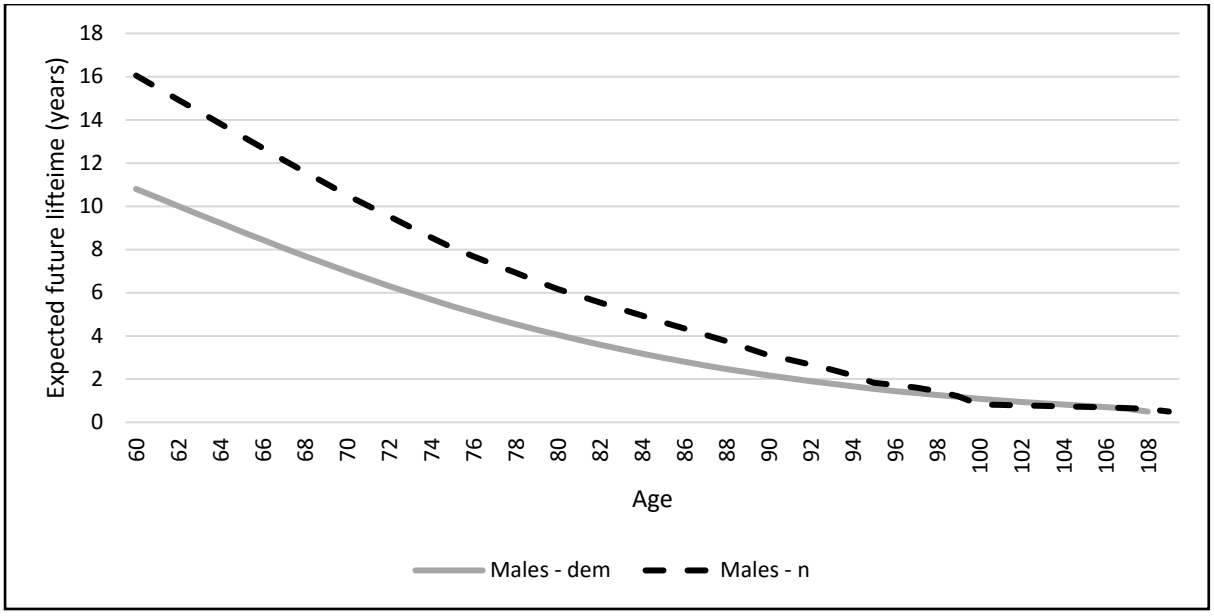
6.1.3 The costs for females are significantly higher than for males and this is due to three factors. Firstly, there is a difference in mortality. The force of mortality for males, without the adjustment for being in the demented state, is around twice that of females in the age band 65–74 and decreases to a ratio of about 1.5 at age 85. Due to high forces of mortality, males are more likely than females to die than to transition into the demented state. Whilst in the demented state, the force of mortality is approximately double the normal rate which causes males to die even faster on average than in the normal state. However, this is doubling in mortality is experienced by both sexes. Secondly, females generally have a higher transition intensity from the normal to the demented state. This results in a higher proportion of females transitioning into the demented state than males. Thirdly, females also transition into institutional care faster than males, this means that they experience higher costs of care for a longer period.

6.1.4 The final result from the model relates to equation (5) – the EPV of costs from age 60 which takes into account the probability of transitioning into the demented state (i.e. the condition that an individual enters into the demented state at a particular age is dropped). Using the notation from equation (5), the EPV (Cost of care) for a female is approximately R261 000 and for a male, it is approximately R92 000. These values can be viewed as the required contribution to a pool at age 60 which indemnifies an individual against the costs of care that may arise due to being in the demented state. The pool will be sustainable if there are a sufficiently large amount of people contributing to the pool.

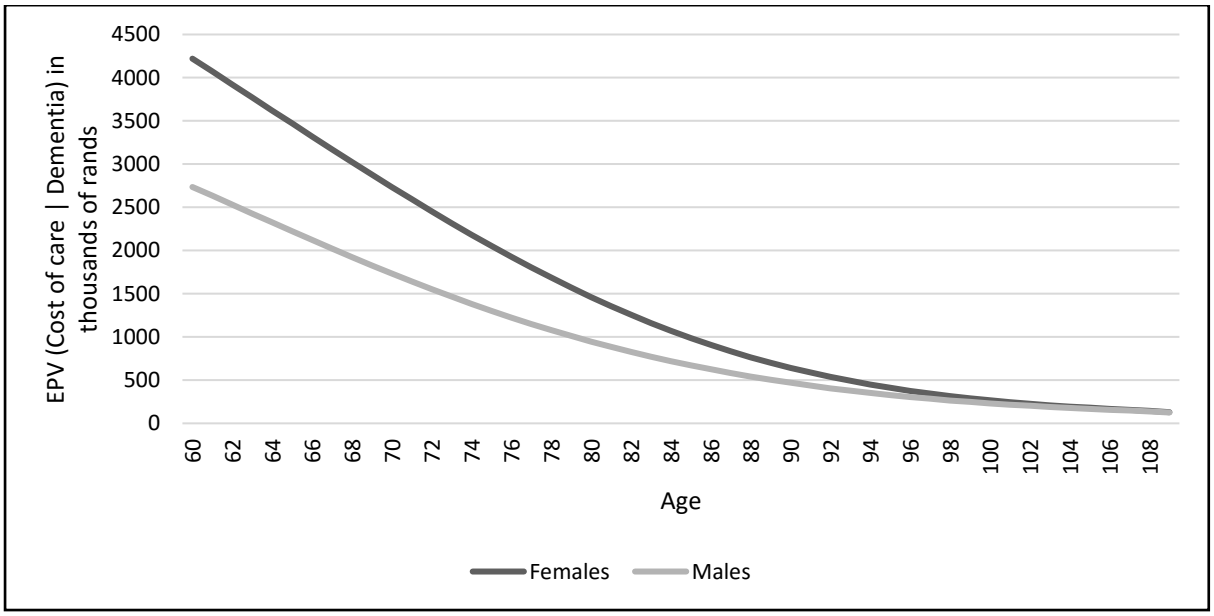


GRAPH 1. Expected future lifetime in the demented state of males and females

13 Steven Nathan. Retirement saving tips the industry doesn't want you to know. http://mfs.holdings/opinion-retirement-savings-tips-the-industry-doesnt-want-you-to-know/, accessed 5 February 2017



GRAPH 2. Expected future lifetime of males in the normal and demented states



GRAPH 3. The expected cost of care in dementia given dementia occurs at specific ages

6.2 Sensitivity Analysis

6.2.1 Sensitivity analysis can be defined as the method of ascertaining how a model's results change and how the model depends on its input parameters (Saltelli, Tarantola & Chan, 1999). Another purpose of sensitivity analysis is to assess the risks that are present and to estimate the impact of reality being different from a model's assumptions (Frey & Patil, 2002). A mathematical method of performing sensitivity analysis, as suggested by Frey & Patil (2002), is to vary the values of model inputs which would represent the range of possible inputs. Then the model results are calculated for each of those different parameter values and the difference in model output is analysed (Frey & Patil, 2002). Therefore, to analyse the sensitivity of the model output to its input variables, the values for the discount rate and the time to institutionalisation will be changed and the output analysed for both males and females.

6.2.2 Butler & Van Zyl (2012) obtained a range of values for the discount rate by setting the pessimistic assumption to be the best-estimate rate minus 1% and the optimistic

assumption to be the best-estimate rate plus 1%. Therefore, a pessimistic rate of 8.7% a year, a best-estimate of 9.7% a year and an optimistic rate of 10.7% a year will be used for the sensitivity analysis of the model to the discount rate used. A similar approach will be used to analyse the sensitivity of the model's results to the time to institutionalisation in the demented state. However, the pessimistic assumption will be that it takes one year less to be institutionalised than the best-estimate assumption, while the optimistic assumption will be that it takes one year longer to be institutionalised after the individual has entered the demented state for the first time. This is because a decrease in the time to institutionalisation will increase the amount of time the individual is expected to be incurring the higher cost of R300 000 a year. Therefore, for males, the pessimistic assumption will be that it takes three years for a male to be institutionalised, the best-estimate assumption will be that it takes four years to be institutionalised and the optimistic assumption will be that it takes five years to be institutionalised. Likewise, for females, the pessimistic assumption, best-estimate assumption and optimistic assumption will be two years, three years and four years respectively for time to institutionalisation. Tables 1 and 2 show the model output for the EPV (Cost of care) which corresponds to equation (5) for both males and females. These figures are rounded off to the nearest R1 000.

TABLE 1. Sensitivity analysis for male EPV (Cost of care)

Discount rate – assumption strength	Time to institutionalisation – assumption strength		
	Optimistic	Best-Estimate	Pessimistic
Optimistic	R 73 000	R 76 000	R 79 000
Best-Estimate	R 88 000	R 92 000	R 96 000
Pessimistic	R108 000	R113 000	R118 000

TABLE 2. Sensitivity analysis for female EPV (Cost of care)

Discount rate – assumption strength	Time to institutionalisation – assumption strength		
	Optimistic	Best-Estimate	Pessimistic
Optimistic	R201 000	R210 000	R221 000
Best-Estimate	R250 000	R261 000	R275 000
Pessimistic	R312 000	R326 000	R343 000

6.2.3 From the tables above, the discount rate has the larger effect on the results of the model. This can be seen by focusing on the best-estimate for both the time to institutionalisation and discount rate of the model in the tables above. The result increases by 23% for males and 25% for females for a 1% decrease in the discount rate, which corresponds to the best-estimate of time to institutionalisation, and the pessimistic assumption for the discount rate. This can be compared to an increase in the result by 4% for males and 5% for females when the time to institutionalisation is reduced by a year, which corresponds to the pessimistic assumption for the time to institutionalisation and the best-estimate for the discount rate. This shows that the model is highly sensitive to changes in the discount rate and far less sensitive to changes in the time to institutionalisation.

6.3 Limitations

6.3.1 Data Limitations

6.3.1.1 The mortality data stops at age 110 for both males and females. This is a limitation because data up until a later age would help to provide results for a larger range of ages. This would provide a more complete estimate of the costs of care due to dementia.

6.3.1.2 The adjustments to mortality for dementia are sourced from a study conducted in Great Britain as data from South Africa could not be found. It would be more relevant to use

a study conducted in South Africa to match the South African mortality data being used, when such studies have been done in South Africa.

6.3.1.3 The incidence rates of dementia are from a foreign country, which may not be suitable to the South African mortality data used in the model. The incidence rates were unstable, particularly for males, as discussed in section 4.3. The range of incidence data was insufficient such that even if mortality data was available, the incidence rates lacked sufficient data in the higher age ranges.

6.3.2 Model Deficiencies

6.3.2.1 The discount rate and medical inflation are both deterministic in the model. Since the rates act in different directions in the model, it would be better to model the effect of the discount rate after accounting for medical inflation in a stochastic manner. This would allow for the results to be more representative of the possible future in South Africa as this would take into account any variations and uncertainties in the discount rate.

6.3.2.2 The structure of the Markov chain in the model does not accurately model dementia. In this Markov chain, dementia is modelled as not allowing for improvements since an individual cannot transition back into the normal state after entering the demented state. Individuals can make improvements in their cognition once they have been diagnosed with dementia (Gow & Gilhooly, 2003).

6.3.2.3 The multi-state model developed by Commenges & Joly (2004) incorporates five different states when modelling dementia, which are: healthy, demented, institutionalised (for reasons other than dementia), demented and institutionalised, and dead. The multi-state model in this paper has excluded the institutionalised state, and then assumed a deterministic transition from home-based care to institutionalised care. This is a significant simplification, which could be improved on if there was sufficient data to calculate transition probabilities from home-based care to institutionalised care.

6.3.2.4 In the model, the medical costs while in the demented state are modelled as a flat cost, which only increases when the individual is moved to institutional care (other than increasing by medical inflation). A more realistic assumption would be for the medical costs to be a function of both the individual's age and how long it has been since the individual first experienced dementia. This is because an individual's health care expenditure should increase with their age as suggested by Zweifel, Felder & Meiers (1999).

7. CONCLUSION

7.1 As discussed in section 3, there are various factors which affect whether a person experiences cognitive decline. The first factor is the age-related decline in cognitive ability which can be viewed as the decline in cognitive ability due to ageing. Fluid intelligence (reasoning ability) declines around age 20 and crystallised intelligence (knowledge acquired) declines around age 40. The second factor is the extent to which an individual uses his or her cognition or the use it or lose it hypothesis. The less a person uses their cognition, the more likely the individual is to experience cognitive decline. The effect of retirement on cognitive abilities can be viewed as a combination of the age-related effect and the use it or lose hypothesis.

7.2 In the demented state, which is a form of cognitive decline, a person experiences costs relating to care. A continuous time Markov chain was constructed to estimate these costs. The Markov chain had three states: normal, demented and dead. The probabilities of transitioning between the different states were then calculated. The complete expected future lifetime of an individual was estimated using the probabilities that were derived from the Markov chain. It

was observed that females survive longer while in the demented state than males. Also, the expected future lifetime in the normal state is higher relative to the expected future lifetime in the demented state for both sexes. However, the difference between the expected future lifetimes decreases towards the higher ages.

7.3 Within the demented state, a person could either be receiving home-based care or institutional care. It was found that the EPV of costs for a male who enters the demented state at age 60 is approximately R3 220 000 and R4 940 000 for females. As very few people in South Africa save enough for retirement, these increased costs would exacerbate their lack of resources. The EPV (Cost of care), as defined in equation (5), is calculated as approximately R261 000 for females and R96 000 for males. This is viewed as the contribution required at the age of 60 to enter the pool which will cover all costs of care due to dementia in retirement, as discussed in section 6.1.4.

7.4 There are, however, problems with the model. Some of the problems include: not all the data that was used to parametrise the model is South African; some of the incidence rates were unstable; and the model does not allow an individual to experience improvements in cognitive health. These limitations should be considered when using the results of the paper.

7.5 Altogether, the paper discussed the various factors influencing cognition, developed a model of transitioning from a normal state to a demented state or to a dead state and then determined the EPV of costs that relate to being in dementia. The discussion of the various factors influencing cognitive decline provides awareness of it and the estimates of the EPV of future costs could allow for better financial planning in retirement. This paper can therefore be used to aid future research into the costs relating to dementia and possibly other forms of cognitive decline in retirement.

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APPENDIX A—SOUTH AFRICAN PENSIONER MORTALITY

TABLE A.1. South African pensioner mortality for males and females for ages 55 to 110.

Male				Female			
Age x	$\mu_{x+0.5}$	Age x	$\mu_{x+0.5}$	Age x	$\mu_{x+0.5}$	Age x	$\mu_{x+0.5}$
55	0.024213	83	0.11137	55	0.010797	83	0.068665
56	0.024555	84	0.11988	56	0.010937	84	0.075840
57	0.025005	85	0.12906	57	0.011140	85	0.083777
58	0.025569	86	0.13896	58	0.011408	86	0.092519
59	0.026255	87	0.14964	59	0.011743	87	0.102117
60	0.027071	88	0.16116	60	0.012146	88	0.112635
61	0.028024	89	0.17360	61	0.012623	89	0.124146
62	0.029124	90	0.18703	62	0.013178	90	0.136737
63	0.030381	91	0.20154	63	0.013818	91	0.150498
64	0.031806	92	0.21720	64	0.014551	92	0.165529
65	0.033409	93	0.23412	65	0.015384	93	0.181936
66	0.035203	94	0.25240	66	0.016328	94	0.199831
67	0.037201	95	0.27215	67	0.017394	95	0.219335
68	0.039416	96	0.29350	68	0.018595	96	0.240578
69	0.041863	97	0.31657	69	0.019945	97	0.263694
70	0.044557	98	0.34150	70	0.021462	98	0.288831
71	0.047514	99	0.36846	71	0.023164	99	0.316140
72	0.050754	100	0.39759	72	0.025071	100	0.345785
73	0.054293	101	0.42910	73	0.027207	101	0.377945
74	0.058153	102	0.46318	74	0.029598	102	0.412817
75	0.062355	103	0.50009	75	0.032274	103	0.450633
76	0.066922	104	0.54013	76	0.035266	104	0.491661
77	0.071879	105	0.58362	77	0.038612	105	0.536200
78	0.077250	106	0.63090	78	0.042350	106	0.584537
79	0.083065	107	0.68219	79	0.046527	107	0.636838
80	0.089351	108	0.73739	80	0.051191	108	0.692962
81	0.096141	109	0.79585	81	0.056397	109	0.752246
82	0.103468	110	0.85630	82	0.062202	110	0.813441

APPENDIX B—INCIDENCE RATES OF DEMENTIA

The following two tables show the data for the incidence rates of dementia – the incidence rate is calculated as the number of people who enter into the demented state for that age band divided by the number of years of exposure in that age band. The two data sources are Ott et al. (1998) and Corrada et al. (2010).

TABLE B.1. Data from Ott et al. (1998)

Age range	Males	Females
55-59	0.0014	0.0000
60-64	0.0090	0.0012
65-69	0.0080	0.0019
70-74	0.0045	0.0036
75-79	0.0148	0.0178
80-84	0.0251	0.0252
85-89	0.0286	0.0504
90-94	0.0296	0.0683
95+	0.0000	0.1115

TABLE B.2. Data from Corrada et al. (2010)

Age range	Males	Females
90-94	0.1230	0.1290
95-99	0.2050	0.2150
100+	0.5520	0.3660

APPENDIX C—SOUTH AFRICAN MEDICAL INFLATION

TABLE C.1. Year-on-year South African medical inflation from January 2012 to April 2016

Month	Year				
	2012	2013	2014	2015	2016
January	8.90%	8.30%	9.20%	8.50%	8.90%
February	7.90%	9.50%	8.00%	9.00%	8.70%
March	7.80%	9.50%	8.00%	9.10%	8.80%
April	7.80%	9.30%	8.20%	9.00%	8.70%
May	8.40%	9.30%	8.20%	9.10%	
June	8.30%	9.30%	8.40%	8.90%	
July	8.40%	9.20%	8.50%	8.90%	
August	8.40%	9.20%	8.40%	9.00%	
September	8.40%	9.30%	8.40%	8.90%	
October	8.30%	9.30%	8.50%	8.90%	
November	8.40%	9.20%	8.50%	8.90%	
December	8.40%	9.20%	8.40%	9.00%	

APPENDIX D—TRANSITION PROBABILITY FORMULAE

S = {n, d, dem} is the state space of the Markov chain. G is the generator matrix with the $(i,j)^{th}$ element of the matrix representing the transition intensity from state i to state j; i, j ∈ S. The rows represent the values of state i whilst the columns represent the values of state j.

$$G = \begin{array}{c} \begin{matrix} \quad n \quad & \quad d \quad & \quad dem \end{matrix} \\ \begin{bmatrix} -(\mu_x+\sigma_x) & \mu_x & \sigma_x \\ 0 & 0 & 0 \\ 0 & \nu_x & -\nu_x \end{bmatrix} \end{array} \begin{matrix} n \\ d \\ dem \end{matrix} \tag{6}$$

The labels above and beside the matrix indicate the state that the column or row represent respectively. Using this generator matrix and Kolmogorov's forward equation (Ross, 2014: 389), the transition probabilities from state i to state j are calculated; i, j ∈ S. Below is Kolmogorov's forward equation from Ross (2014):

$${}_tP_x^{(i,j)} = \sum_{k \in S;\, k \neq j} {}_tP_x^{(i,k)} \cdot \lambda_x^{(k,j)} - {}_tP_x^{(i,j)} \cdot \lambda_x^{(j,j)} \tag{7}$$

where:

—${}_tP_x^{(i,j)}$ is the transition probability for a person aged x from state i to state j

—$\lambda_x^{(k,j)}$ is the transition intensity for a person aged x from state k to state j

Since the transition intensities are assumed to be constant in the interval (x, x+1] (as per section 5.2), the value of t in the equations derived below is limited to the interval (0,1]. Multiple transition probabilities can be multiplied if a longer transition interval is needed; for example, ${}_1P_x^{(n,n)} \, {}_1P_{x+1}^{(n,n)} = {}_2P_x^{(n,n)}$. The following are the all the non-zero transition probabilities.

$${}_tP_x^{(n,n)} = e^{-(\mu_x+\sigma_x)t} \tag{8}$$

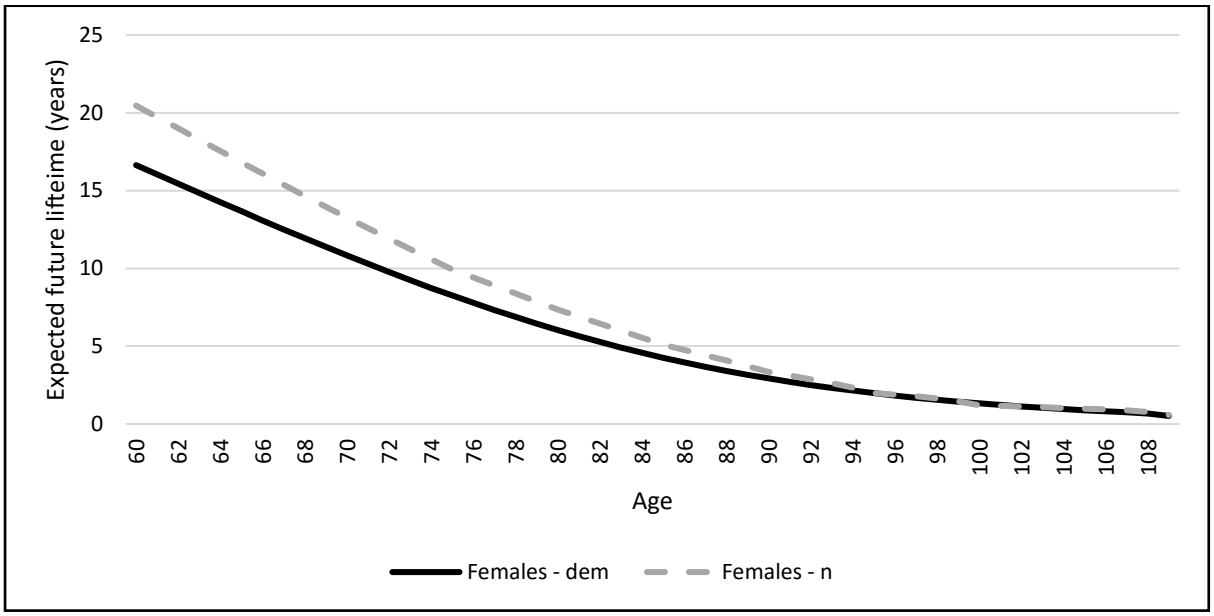
$${}_tP_x^{(n,dem)} = \frac{\sigma_x e^{-\nu_x t}}{\sigma_x+\mu_x-\nu_x}\left[1 - e^{-(\mu_x+\sigma_x-\nu_x)t}\right] \tag{9}$$

$${}_tP_x^{(n,d)} = 1 - \left[{}_tP_x^{(n,n)} + {}_tP_x^{(n,dem)}\right] \tag{10}$$

$${}_tP_x^{(dem,dem)} = e^{-\nu_x t} \tag{11}$$

$${}_tP_x^{(dem,d)} = 1 - e^{-\nu_x t} \tag{12}$$

APPENDIX E—EXPECTED FUTURE LIFETIME OF FEMALES



GRAPH 4. Expected future lifetime of females in the normal and demented states