Evaluating Retirement Strategies: A Utility-Based Approach *

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Abstract. *Retirees need to make two critical financial decisions, namely, the withdrawal rate and the asset allocation of their portfolios. We propose a methodology that retirees, and particularly advisors, could use to make these decisions in an optimal way. We introduce a new variable, the coverage ratio, and a theoretical approach, based on utility. Our approach can be used to make optimal decisions during both the accumulation and the retirement period, but we illustrate it by focusing on the latter, and particularly on the choice of an optimal asset allocation. We find that the strategies selected by our utility-based approach are in general somewhat more aggressive than those selected by the failure rate and other existing approaches*.

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1. Introduction

Individuals saving for retirement need to determine a saving rate, much like retirees need to determine a withdrawal rate. In addition, both need to determine an asset allocation for their portfolios, and decide how the asset allocation should evolve over time. The ultimate issue we address in this article is how to make these choices in an optimal way.

Academics and practitioners have invested considerable time and effort toward deriving measures to evaluate investment strategies. We contribute to this debate by proposing both a new variable, the coverage ratio, and a theoretical approach, based on utility, that aim to help investors, and particularly advisors, determine an optimal strategy.

For the sake of concreteness, we illustrate our approach by focusing on the retirement period, and particularly on the optimal choice among competing asset allocations. Needless to say, our approach is just as valid to choose an optimal withdrawal rate during the retirement period, or a saving rate and asset allocation during the accumulation period.

Many variables have been previously proposed to evaluate investment strategies during retirement. Of these, the most widely used is the failure rate, which aims to capture how often a

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strategy failed to sustain a withdrawal plan through the end of the retirement period. We propose an alternative variable, the coverage ratio, which aims to capture the fraction of the retirement period a strategy was able to sustain a withdrawal plan. For reasons we discuss below, we believe that the coverage ratio is a better variable than the failure rate to evaluate retirement strategies.

But we do not stop there; we do not propose to evaluate competing strategies simply by comparing coverage ratios. We believe that the correct approach should consider the *utility* derived from a strategy's coverage ratio by an individual with a specific level of risk aversion and personal cost of depleting his portfolio too early. The kinked utility function we propose has the desirable property that displays positive utility when a strategy successfully sustained a withdrawal plan, and negative utility when the strategy failed to do so.

Finally, we do not just propose a theoretical framework based on the coverage ratio and a kinked utility function; we also implement our approach empirically. To do so, we use a comprehensive sample of 21 countries and the world market over a 115-year period, and we apply our approach to the selection of the optimal asset allocation, of the 11 we consider, for each country in our sample. We find that the strategies selected by our approach are in general somewhat more aggressive than those selected by the failure rate and other existing approaches

The rest of the article is organized as follows. In section 2, we explore in more detail the issue at stake, first by discussing the failure rate and other variables used to evaluate retirement strategies; then by introducing a new variable, the coverage ratio; and finally by proposing a utility-based approach. In section 3, we apply our framework empirically and determine the optimal asset allocation for each of the markets in our sample. Finally, in section 4, we provide an assessment. An appendix with tables concludes the article.

2. The Issue

In retirement research, the failure rate refers to the number of retirement periods in which a strategy failed, relative to the total number of (simulated or historical) retirement periods considered. In this context, a strategy fails when it depletes a portfolio too early, thus not being able to sustain a withdrawal plan during an individual's entire retirement period.

Although ubiquitous in the literature, the failure rate is not the only variable that can be used to evaluate retirement strategies. We start this section by discussing other variables that have been proposed in the literature. Then we introduce a new variable, the coverage ratio, which has several advantages over the failure rate. And finally we propose a utility-based approach, based on the coverage ratio, that aims to help investors, and particularly advisors, determine an optimal retirement strategy.

2.1. Existing Approaches

The failure rate can be traced back to the groundbreaking article by Bengen (1994), in which he aims to determine a safe withdrawal rate. He finds that an initial withdrawal rate of 4%, with subsequent withdrawals adjusted by inflation, is safe in the sense that (historically) it never depleted a portfolio (failed) before 30 years, which he considered a minimum requirement for portfolio longevity. From that point on, the failure rate became the standard tool used to evaluate retirement strategies.

That said, the failure rate does have several shortcomings (see Milevsky, 2016), which led academics and practitioners to propose alternatives. Blanchett (2007) argues that a strategy's probability of success is important, but so is the variability of the underlying portfolio. For this reason, he proposes to evaluate retirement strategies with the *success-to-variability ratio*, which accounts for both the probability of success (1 minus the probability of failure) and the variability of the portfolio during the retirement period.

Das et al (2010) integrate mean-variance portfolio theory and behavioral portfolio theory into a mental accounting framework; they consider a portfolio as a collection of mental account subportfolios and define risk (failure) as the probability of not achieving the goal set for each mental account. Brunel (2015) and Parker (2016) start by defining an individual's personal goals and view risk (failure) as the probability of not achieving those goals.

Estrada (2017) addresses the issue that although the failure rate captures how often a strategy failed, it does not capture by how much it did so. He therefore introduces the variable *shortfall years*, which aims to complement the failure rate by measuring the average number of years a strategy failed to support withdrawals over all the retirement periods in which it failed.

Estrada (2018*a*, 2018*c*) further proposes to change the perspective from failure to success and introduces the variable *years sustained*, which measures the average number of years a strategy sustained withdrawals both when it failed and when it succeeded. He also introduces the variables *risk-adjusted success* and *downside risk-adjusted success*, which relate a strategy's years of withdrawals sustained to two different measures of risk.

Finally, Suarez et al (2015) and Clare et al (2017) propose to evaluate retirement strategies with the *perfect withdrawal amount*, which aims to determine the constant real withdrawal that would leave exactly the desired bequest at the end of the retirement period. In this framework, the optimal strategy is the one with the highest perfect withdrawal amount simply because it enables the highest standard of leaving. Slight variations of this framework were proposed by Blanchett et al (2012), who consider the *sustainable spending rate*; and Miller (2016) and Estrada (2018*b*) who consider the *maximum withdrawal rate*.

Using the same sample, which we describe in more detail in section 3, Exhibit 1 shows the asset allocation selected for each country in our sample by some of the variables previously proposed in the literature, including the failure rate (*F*), shortfall years (S_Y), risk-adjusted success (RAS), downside risk-adjusted success (D-RAS), and the maximum withdrawal rate (MWR). The figures in the exhibit show the proportion of stocks in the optimal asset allocation, the rest of the portfolio being invested in bonds.

Exhibit 1: Optimal Choice – Some Existing Approaches

This exhibit shows the asset allocation selected by the failure rate (*F*), shortfall years (S_Y), risk-adjusted success (RAS), downside risk-adjusted success (D-RAS), and the maximum withdrawal rate (MWR). All figures show the proportion of stocks in the portfolio (in %), the rest being allocated to bonds. The 11 asset allocations considered range from 100 (all stocks) to 0 (no stocks), with nine allocations (90, 80, …, 20, 10) in between. All strategies are evaluated over 86 rolling 30-year retirement periods between 1900-1929 and 1985-2014; a starting capital of $1,000; a 4% initial withdrawal rate (except in the MWR case); subsequent annual withdrawals adjusted by inflation; and annual rebalancing. The data is described in Exhibit A1 in the appendix.

Country	***F***	S_Y	**RAS**	**D-RAS**	**MWR**
Australia	100	60	100	90	100
Austria	80 & 60	90	80	70	50
Belgium	100	70	30	100	90
Canada	90-70	60	60	60	100
Denmark	90	70	90	70	100
Finland	100	0	0	100	100
France	100	100	100	100	100
Germany	70-50	100	0	100	90
Ireland	100	40	100	100	100
Italy	100	80	100	100	100
Japan	50	0	10	100	90
Netherlands	100	50	70	70	90
New Zealand	100-90	80	70	80	100
Norway	100-90	60	90	80	100
Portugal	50	70	50	70	90
South Africa	100-80	60	80	100	100
Spain	100	30	0	90	100
Sweden	60 & 40	30	40	40	100
Switzerland	50-40	0	10	80	80
UK	100	90	100	100	100
USA	100-70	80	70	80	100
World	100	100	100	100	100
Average	**81**	**60**	**61**	**85**	**95**

Exhibit 1 clearly shows that, for any given country, different variables may lead to the selection of very different strategies. It also shows that, on average, some variables tend to select strategies far more aggressive than others. Across all the markets in our sample, the failure rate selects strategies with an average allocation of 81% in stocks and 19% in bonds.

2.2. The Coverage Ratio

We introduce in this section the new variable we propose, the coverage ratio; and in the next section the approach we propose, based on utility. Let *f* be a variable that takes a value of 1 in a retirement period in which a strategy failed and 0 otherwise. Then, the *failure rate* (*F*) is formally defined as

$$F = \left(\frac{1}{T}\right) \cdot \sum_{t=1}^{T} f_t \tag{1}$$

where T is the number of (historical or simulated) retirement periods considered and t indexes retirement periods, both typically measured in years.

Let Y_t be the number of years of inflation-adjusted (real) withdrawals sustained by a strategy, both during and after the retirement period, and L be the length of the retirement period considered. Then we define the *coverage ratio* in retirement period t (C_t) as

$$C_t = Y_t/L \tag{2}$$

By definition, C<1 indicates that the strategy depleted the portfolio before the end of the retirement period; C>1 indicates that the strategy sustained withdrawals through the entire retirement period and left a bequest behind; and C=1 indicates that the strategy sustained withdrawals exactly through the end of the retirement period with no bequest left behind.

To illustrate the intuition behind our coverage ratio, consider (as we do later in our empirical section) a $1,000 portfolio at the beginning of retirement, a 4% initial withdrawal rate, subsequent annual withdrawals adjusted by inflation, and a 30-year retirement period. This set-up yields an initial withdrawal of $40, followed by 29 withdrawals of $40 in inflation-adjusted dollars. Consider also three scenarios: In the first, the portfolio is depleted after 24 years, six years short of the end of the retirement period; in the second, the portfolio is depleted after 30 years, having sustained 30 years of withdrawals but leaving no bequest behind; and in the third, the portfolio sustained 30 years of withdrawals and left behind a bequest of $240 in real dollars.

Then, by definition, in the first, second, and third scenarios our Y_t variable would be 24, 30, and 36; and our coverage ratios would be C=0.80, C=1, and C=1.2. Note that in the third scenario the strategy sustained 30 years of withdrawals during the entire retirement period and left behind a bequest of $240 in real dollars, which amounts to six additional years of $40 withdrawals in real dollars.

The appeal of our coverage ratio is that it consolidates in one variable both the frequency and the magnitude of success or failure. For this reason, it provides more information than the failure rate (which measures only the frequency of failure) and shortfall years (which measures only the magnitude of failure). Furthermore, although the failure rate does not distinguish between two strategies that succeeded but left behind very different bequests, the coverage ratio (which would increase with the size of the bequest) does.

That said, our coverage ratio and the failure rate are clearly related. If F=0, then it must be the case that $C_t \geq 1$ for all t, implying that the strategy sustained withdrawals for at least 30 years in every retirement period considered; if F>0, then it must be the case that C_t<1 for at least some t, implying that at least in one retirement period the strategy fell short; and if F=1, then it must be the case that $C_t < 1$ for all t, implying that the strategy failed in every period.

We calculate this coverage ratio for each strategy, retirement period, and market we consider. Given that, as we discuss in more detail below, our analysis covers 22 markets, 11 strategies (asset allocations), and 86 retirement periods, our analysis and conclusions are based on evidence from 20,812 coverage ratios.

2.3. A Utility-Based Approach

The evaluation of retirement strategies typically involves comparing the failure rate of the strategies considered, and perhaps some additional moments or percentiles of the distribution of failure rates. We believe our coverage ratio provides a more comprehensive assessment of a strategy's success or failure than the failure rate. However, we do not propose to calculate the average coverage ratio across all the retirement periods considered for a strategy, and compare it to that of a competing strategy. Although doing so would account for the success or failure of different strategies, it would fail to consider risk. Rather, we propose to consider coverage ratios together with the utility a retiree derives from them.

To that purpose, we propose a utility function in which as the coverage ratio increases above 1, a retiree's utility increases at a decreasing rate, thus implying risk aversion. This notion of utility was first proposed by Daniel Bernoulli in 1738 and is a widely accepted description of preferences throughout the finance literature.

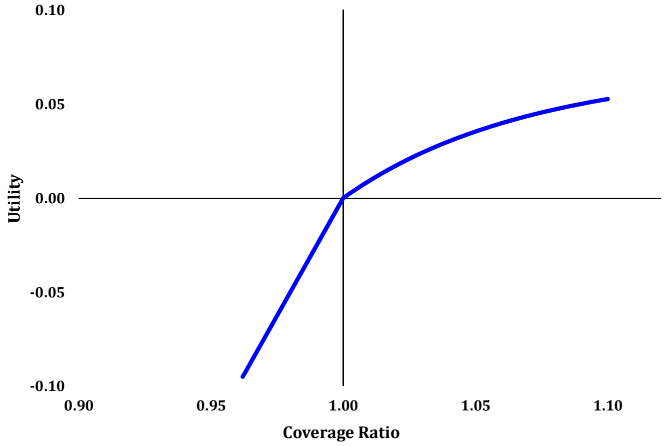
We believe that this is a proper description of a retiree's utility as long as the coverage ratio is higher than 1. However, when the coverage ratio falls below 1, thus implying a strategy's failure to sustain withdrawals through the entire retirement period, it makes sense to assume that a retiree would experience a steep decline in utility. Therefore, we propose a kinked utility function given by the expression

$$U(C) = \frac{C^{1-\gamma} - 1}{1-\gamma} \qquad \text{for } C \geq 1$$

$$U(C) = \frac{1^{1-\gamma} - 1}{1-\gamma} - \lambda(1 - C) \qquad \text{for } C < 1 \tag{3}$$

where U denotes utility; γ is the coefficient of risk aversion, which determines the curvature of the slope when $C > 1$; and λ is a linear penalty coefficient when $C < 1$. Exhibit 2 depicts the kind of utility function we propose, which is similar to that used by Adler and Kritzman (2007) and Czasonis et al (2018).

Exhibit 2: A Kinked Utility Function



As both the expression and the exhibit show, this utility function has a very appealing property, namely, that the kink is located at a coverage ratio of 1. Therefore, when $C = 1$, implying that a strategy sustained withdrawals during the entire retirement period but left no bequest behind, $U(C) = 0$. When $C > 1$, implying that a strategy sustained withdrawals during the entire retirement period and in addition left a bequest behind, utility increases at a decreasing rate with the size of the bequest. And when $C < 1$, implying that a strategy failed to sustain withdrawals during the entire retirement period, utility falls steeply and linearly with the size of the shortfall.

Note that locating the kink below $C = 1$ would imply negative utility even when a strategy did not fail. This would imply an unintended penalty, and one that is difficult to interpret. We believe that locating the kink exactly at $C = 1$ is a very clean solution. Furthermore, we believe it is both plausible and intuitive to assume that when the coverage ratio is higher than 1, utility increases at a decreasing rate with the size of the bequest; and when it is lower than 1, utility falls more steeply with the size of the shortfall. In other words, a retiree is less happy with a surplus than he is unhappy with a shortfall of the same amount.

It should be clear from (3) that γ determines the curvature of the function when $C > 1$, with higher values indicating that utility increases more slowly as the coverage ratio increases. Furthermore, λ determines the steepness of the function when $C < 1$, with higher values indicating that utility decreases more rapidly as the coverage ratio decreases. In our base case scenario we consider $\gamma = 0.9999$ (essentially logarithmic utility for positive coverage ratios) and $\lambda = 10$. While we are confident on the form of the utility function we propose, we understand that there are no universally accepted values for γ and λ. We believe that our choice of parameters provides sensible results, but we also perform a sensitivity analysis on these two parameters to provide a broader perspective.

3. Evidence

This section discusses the global evidence based on 21 countries and the world market over the 115 years between 1900 and 2014. We start by describing our sample and basic methodology; then we discuss the results of a preliminary analysis based on our coverage ratio but not on a utility-based framework; and finally we discuss the optimal selection of strategies based on the coverage ratio and utility function discussed in the previous section.

3.1. Data and Methodology

The sample we consider is the Dimson-Marsh-Staunton database, described in detail in Dimson, Marsh, and Staunton (2002, 2016). It contains annual returns for stocks and long-term government bonds over the 1900-2014 period for 21 countries and the world market. Returns are real (adjusted by each country's inflation rate), in local currency (except for the world market, in dollars), and account for both capital gains/losses and cash flows (dividends or coupons). Exhibit A1 in the appendix summarizes some characteristics of all the series of stock and bond returns in the sample.

The analysis is based on a $1,000 portfolio at the beginning of retirement, a 4% initial withdrawal rate, annual inflation-adjusted withdrawals, and a 30-year retirement period.[1] At the beginning of each year the annual withdrawal is made, the portfolio is then rebalanced to the target asset allocation for the year, and then it compounds at the observed return of stocks and bonds for that year. This process is repeated at the beginning of each year during the 30-year retirement period, at the end of which the portfolio has a terminal wealth or bequest that may be positive or 0. The first 30-year retirement period considered is 1900-1929 and the last one is 1985-2014, for a total of 86 rolling (overlapping) periods.

The analysis considers 11 stock-bond allocations ranging from 100 (all stocks, no bonds) to 0 (no stocks, all bonds), with nine allocations (90, 80, …, 20, 10) in between, all indicated by the proportion of stocks in the portfolio, with the balance allocated to bonds. The ultimate goal of our analysis is to find the optimal asset allocation for each country based on the coverage ratio and utility-based framework we propose.

3.2. Preliminary Analysis

As our previous discussion makes clear, the coverage ratio needs to be calculated for every retirement period evaluated. This means that given the 86 retirement periods between

[1] Changing the initial withdrawal rate would change all our results but none of the conclusions we draw from the analysis. As already mentioned, our main goal is to introduce a methodology that can be used to evaluate retirement strategies, and to that purpose we just focus on the choice among different asset allocations. Sexauer et al (2012), Sexauer and Siegel (2013), Cook et al (2015) and Waring and Siegel (2015) discuss other spending rules.

1900-1929 and 1985-2014 in our sample, we need to calculate 86 coverage ratios for each of the 11 strategies and 22 markets we consider. This process yields a distribution of the coverage ratio for each asset allocation and country (hence 242 distributions), and we start by focusing on three parameters of these distributions, namely, the mean (AM), the standard deviation (SD), and the semideviation with respect to 1 (SSD).[2]

Exhibit 3 shows two ratios based on these three parameters, which can be thought of as measures of risk-adjusted performance. Panel 1 shows the ratio between AM and SD, and panel 2 the ratio between AM and SSD. The selected asset allocation for each country based on the highest value of these two ratios across the 11 asset allocations we consider is highlighted in the exhibit.

Exhibit 3 clearly shows that the optimal asset allocation varies substantially from market to market and between the two ratios we consider. The exhibit also shows that the AM/SSD ratio tends to select more aggressive strategies than the AM/SD ratio. This is largely due to the fact that large deviations of the coverage ratio above 1 tend to increase SD but not SSD; therefore, unlike the standard deviation, the semideviation does not penalize (aggressive) strategies that leave large bequests.

Estrada (2018*c*) discusses this issue in more detail, but for our current purposes consider Belgium, an extreme case in which the AM/SD ratio selects an allocation with no stocks, and the AM/SSD ratio selects one with no bonds. The standard deviation of coverage ratios (not reported) decreases steadily between the 100 and the 0 stock allocations, from 2.22 to 0.96; the semideviation (not reported), on the other hand, increases steadily from 0.27 to 0.36. Allocations with high proportion of stocks result in some very large coverage ratios (8, 9, and higher), which increase the standard deviation but have no impact on the semideviation.

We do not pursue this line of inquiry much further. Although these two ratios account for both the success/failure and risk of each strategy, we believe that a proper evaluation should involve a utility function that incorporates a retiree's risk aversion and a penalty for strategies that fail to sustain a withdrawal plan through the entire retirement period.[3] That is the issue we consider in the next section.

[2] This semideviation measures volatility below (but not above) 1, a benchmark we chose simply because it is the coverage ratio that divides success from failure.
[3] Furthermore, note that measures like AM/SD and AM/SSD rest on the assumption of unlimited borrowing and lending at the risk-free rate, which may not be possible for (or desirable to) many retirees.

Exhibit 3: Coverage Ratio – Preliminary Analysis

This exhibit shows two measures of risk-adjusted performance, for the 11 asset allocations considered, ranging from 100 (all stocks, no bonds) to 0 (no stocks, all bonds), with nine allocations (90, 80, …, 20, 10) in between, the rest being allocated to bonds. Panel 1 shows the AM/SD ratio, where the numerator is the arithmetic mean (AM) and the denominator the standard deviation (SD); panel 2 shows the AM/SSD ratio, where the denominator is the semideviation with respect to 1 (SSD). AM, SD, and SSD are calculated for coverage ratios over the 86 rolling 30-year retirement periods considered between 1900-1929 and 1985-2014. All strategies consider a starting capital of $1,000; a 4% initial withdrawal rate; subsequent annual withdrawals adjusted by inflation; and annual rebalancing. The data is described in Exhibit A1 in the appendix.

Panel 1	**100**	**90**	**80**	**70**	**60**	**50**	**40**	**30**	**20**	**10**	**0**
Australia	1.51	1.55	1.56	1.55	1.51	1.46	1.40	1.33	1.27	1.24	1.24
Austria	1.08	1.09	1.09	1.08	1.07	1.06	1.06	1.07	1.10	1.13	1.18
Belgium	0.83	0.84	0.85	0.87	0.89	0.91	0.94	0.98	1.02	1.08	1.14
Canada	1.74	1.91	2.02	2.03	1.94	1.78	1.61	1.45	1.30	1.18	1.08
Denmark	1.29	1.21	1.15	1.11	1.07	1.04	1.03	1.01	1.01	1.01	1.01
Finland	0.77	0.78	0.79	0.80	0.81	0.82	0.85	0.89	0.96	1.08	1.28
France	0.78	0.77	0.77	0.77	0.78	0.79	0.81	0.83	0.86	0.89	0.93
Germany	0.48	0.54	0.61	0.69	0.79	0.89	0.99	1.08	1.18	1.25	1.33
Ireland	0.89	0.90	0.90	0.90	0.90	0.89	0.89	0.89	0.91	0.94	0.98
Italy	1.13	1.11	1.07	1.04	1.04	1.03	1.03	1.04	1.05	1.06	1.08
Japan	0.62	0.69	0.79	0.92	1.08	1.27	1.46	1.59	1.61	1.53	1.41
Netherlands	1.05	1.06	1.07	1.09	1.11	1.13	1.18	1.24	1.31	1.42	1.54
New Zealand	2.14	2.15	2.09	1.98	1.86	1.75	1.65	1.58	1.53	1.50	1.49
Norway	0.80	0.82	0.84	0.87	0.90	0.94	0.99	1.04	1.09	1.13	1.16
Portugal	0.93	0.92	0.92	0.93	0.97	1.04	1.13	1.23	1.31	1.34	1.30
South Africa	1.65	1.71	1.76	1.80	1.83	1.86	1.88	1.91	1.96	2.02	2.11
Spain	0.65	0.67	0.69	0.73	0.77	0.82	0.89	0.97	1.08	1.22	1.39
Sweden	0.85	0.84	0.84	0.84	0.86	0.88	0.93	0.99	1.07	1.17	1.28
Switzerland	1.52	1.59	1.67	1.75	1.84	1.92	2.01	2.09	2.15	2.17	2.16
UK	1.46	1.45	1.42	1.38	1.34	1.29	1.25	1.22	1.20	1.20	1.21
USA	1.60	1.77	1.86	1.86	1.78	1.67	1.55	1.44	1.36	1.30	1.27
World	1.81	1.85	1.81	1.72	1.60	1.49	1.39	1.31	1.25	1.20	1.16
Panel 2	**100**	**90**	**80**	**70**	**60**	**50**	**40**	**30**	**20**	**10**	**0**
Australia	63.35	57.28	51.10	42.26	33.19	23.89	15.47	10.25	7.19	5.02	3.79
Austria	3.63	4.24	4.56	4.60	4.41	4.09	3.70	3.30	2.88	2.48	2.10
Belgium	6.73	6.57	6.38	6.09	5.65	5.19	4.73	4.28	3.84	3.42	3.03
Canada	231.42	N/A	N/A	N/A	317.49	54.69	24.40	15.24	10.62	7.28	5.41
Denmark	91.06	143.99	173.92	236.15	179.20	92.52	53.44	32.33	20.47	14.00	10.17
Finland	14.42	12.93	11.44	9.70	8.20	6.82	5.58	4.49	3.60	2.87	2.29
France	7.30	7.00	6.67	6.41	6.10	5.80	5.43	5.02	4.66	4.19	3.78
Germany	6.96	6.27	5.65	5.12	4.60	4.15	3.77	3.34	2.98	2.62	2.29
Ireland	17.36	15.90	14.17	12.67	11.13	9.54	8.06	6.60	5.34	4.42	3.72
Italy	3.32	3.17	3.06	2.93	2.84	2.69	2.56	2.41	2.26	2.06	1.87
Japan	13.31	11.82	10.35	9.07	7.88	6.84	5.88	5.08	4.40	3.78	3.22
Netherlands	26.26	29.02	30.26	31.32	27.22	21.19	15.74	11.12	7.95	5.87	4.46
New Zealand	N/A	N/A	743.29	226.69	112.85	47.14	24.46	14.40	9.53	6.72	5.00
Norway	10.75	11.24	11.22	10.91	10.23	9.37	8.17	7.14	5.94	4.86	3.93
Portugal	7.85	8.18	8.61	8.60	8.23	7.42	6.52	5.52	4.66	3.95	3.36
South Africa	273.31	233.90	155.63	130.51	84.28	52.29	29.80	15.57	9.04	5.82	3.96
Spain	7.00	7.16	7.04	6.93	6.71	6.45	6.11	5.64	5.04	4.37	3.77
Sweden	23.00	23.40	23.89	25.20	27.93	30.44	31.59	26.69	15.96	9.96	6.73
Switzerland	11.02	11.22	11.53	11.48	11.23	10.96	10.65	10.28	9.50	8.65	7.51
UK	78.79	62.31	41.82	27.10	19.00	14.16	10.87	8.28	6.29	4.86	3.91
USA	87.62	142.53	195.62	158.45	107.61	67.72	39.89	22.58	13.01	8.07	5.40
World	35.21	28.36	22.40	17.30	13.55	10.87	8.87	7.16	5.89	5.10	4.32

3.3. Results from Our Utility-Based Approach

In order to implement the utility-based framework we propose, we take the following steps for each of the 11 strategies and 22 markets we consider. First, we calculate the coverage ratio for each of the 86 rolling 30-year retirement periods in our sample. Then we calculate the utility a retiree derives from each coverage ratio; to do so we use the utility function in (3) and assume $\gamma = 0.9999$ and $\lambda = 10$ in our base case. Finally, we calculate expected utility by averaging the utilities in the previous step across the 86 retirement periods. This process yields a figure that summarizes the average utility a retiree perceives from a given strategy in a given market. Exhibit 4 summarizes the results of our analysis; the selected asset allocation for each market, based on the highest average utility, is highlighted.

Exhibit 4: Coverage Ratio – Utility-Based Approach – Base Case

This exhibit shows the average utility for the 11 asset allocations considered, ranging from 100 (all stocks, no bonds) to 0 (no stocks, all bonds), with nine allocations (90, 80, …, 20, 10) in between, the rest being allocated to bonds. The utility function is given by expression (3) in the text, for γ=0.9999 and λ=10. All strategies are evaluated over 86 rolling 30-year retirement periods between 1900-1929 and 1985-2014; a starting capital of $1,000; a 4% initial withdrawal rate; subsequent annual withdrawals adjusted by inflation; and annual rebalancing. The data is described in Exhibit A1 in the appendix.

Country	**100**	**90**	**80**	**70**	**60**	**50**	**40**	**30**	**20**	**10**	**0**
Australia	1.25	1.13	0.99	0.83	0.62	0.34	−0.07	−0.52	−1.02	−1.64	−2.13
Austria	−2.24	−1.92	−1.79	−1.82	−1.92	−2.06	−2.20	−2.39	−2.59	−2.84	−3.12
Belgium	−1.29	−1.32	−1.35	−1.41	−1.52	−1.62	−1.73	−1.86	−2.01	−2.18	−2.39
Canada	1.04	0.98	0.90	0.81	0.70	0.48	0.20	−0.10	−0.50	−1.09	−1.59
Denmark	0.71	0.72	0.70	0.66	0.62	0.53	0.38	0.18	−0.12	−0.44	−0.80
Finland	−0.55	−0.71	−0.86	−1.06	−1.23	−1.40	−1.62	−1.86	−2.09	−2.38	−2.77
France	−1.21	−1.40	−1.56	−1.67	−1.77	−1.85	−1.95	−2.05	−2.13	−2.28	−2.41
Germany	−1.90	−1.97	−2.07	−2.15	−2.23	−2.31	−2.38	−2.49	−2.59	−2.72	−2.87
Ireland	−0.04	−0.13	−0.25	−0.37	−0.51	−0.73	−1.02	−1.39	−1.79	−2.13	−2.41
Italy	−2.24	−2.33	−2.43	−2.59	−2.68	−2.81	−2.92	−3.04	−3.16	−3.34	−3.54
Japan	−0.79	−0.78	−0.80	−0.83	−0.87	−0.94	−1.06	−1.18	−1.32	−1.52	−1.93
Netherlands	0.36	0.38	0.35	0.32	0.21	0.02	−0.25	−0.59	−0.97	−1.37	−1.74
New Zealand	1.03	0.96	0.87	0.77	0.65	0.46	0.19	−0.18	−0.58	−0.99	−1.50
Norway	−0.51	−0.47	−0.50	−0.57	−0.68	−0.81	−0.99	−1.16	−1.45	−1.78	−2.12
Portugal	−1.22	−1.04	−0.87	−0.77	−0.72	−0.83	−1.05	−1.36	−1.66	−1.95	−2.27
South Africa	1.39	1.25	1.09	0.94	0.76	0.56	0.29	−0.14	−0.60	−1.17	−1.79
Spain	−1.18	−1.11	−1.10	−1.08	−1.10	−1.15	−1.24	−1.37	−1.54	−1.76	−1.98
Sweden	0.26	0.30	0.32	0.33	0.34	0.31	0.28	0.10	−0.23	−0.65	−1.11
Switzerland	−0.34	−0.28	−0.22	−0.21	−0.21	−0.23	−0.25	−0.31	−0.39	−0.53	−0.76
UK	0.90	0.75	0.56	0.33	0.10	−0.12	−0.40	−0.81	−1.32	−1.81	−2.20
USA	1.05	1.00	0.93	0.82	0.69	0.54	0.34	0.06	−0.37	−0.95	−1.52
World	0.62	0.50	0.35	0.15	−0.06	−0.31	−0.58	−0.87	−1.12	−1.33	−1.68

Our approach results in the selection of relatively aggressive strategies, with an average allocation of 91% to stocks and 9% to bonds. In over half of the markets, including the U.S. and the world market, the strategy selected is the most aggressive of those considered; 100% stocks. The most conservative strategy selected, in only two countries (Portugal and Sweden), is a portfolio with 60% in stocks. In two other countries (Spain and Switzerland) the optimal choice consists of 70% in stocks; in one country (Austria) the highest utility corresponds to an allocation of 80% to stocks; and in the rest of the countries the strategy selected consists of 90% in stocks.

A comparison of Exhibits 1 and 4 shows that our utility-based approach and the failure rate select the same strategy in nine of the 22 markets we consider. In six markets the two approaches are consistent, in the sense that the range of allocations selected by the failure rate (which not always selects a unique strategy) contains the selection made by our approach. Finally, in seven markets the two approaches select different strategies, although in three of those cases the selected allocations are within 10 percentage points of each other.

Furthermore, with the sole exception of the Netherlands and Spain (and the partial exception of Norway), our utility-based approach always selects a strategy that is at least as aggressive as that selected by the failure rate. This should not be entirely surprising. For the failure rate, only success or failure matter; in our approach, the size of the bequest, which tends to increase with the allocation to stocks, matters too. On average, the failure rate selects allocations with 81% in stocks, whereas the average allocation to stocks based on our approach is 91%.

3.4. Sensitivity Analysis and Further Discussion

As we already mentioned, we are confident on the shape of the utility function we propose, and we think our choice of parameters for the base case (γ = 0.9999 and λ = 10) is sensible. That said, we explore how sensitive the results we just discussed are to changes in the value of these two parameters.

Although λ is a penalty coefficient specific to our utility function, there is a vast literature that discusses plausible values for the risk aversion coefficient (γ). Gandelman and Hernández-Murillo (2014) estimate this coefficient at the aggregate level for 75 countries and obtain a range between 0 and 3; a value in the vicinity of 1 for the vast majority of countries; and a cross-sectional average of 0.98. They conclude that their overall results support the use of a logarithmic utility function, which is essentially what we do for coverage ratios higher than 1. Furthermore, Thomas (2016) argues that the UK Treasury recommends the use of a risk-aversion coefficient equal to 1.

Exhibits A2 through A7 in the appendix report the results of a sensitivity analysis on the γ and λ coefficients of our utility function. The first three exhibits (A2 through A4) consider changes in the value of the risk aversion coefficient from 0.9999 in the base case to 0, 0.5, and 2. Predictably, higher values of γ (greater risk aversion), lead to more conservative strategies. Still, we find that the most conservative strategies selected consist of allocations with at least 40% in stocks.

Exhibits A5 through A7 report the results of a sensitivity analysis on the penalty coefficient that applies when the underlying strategy fails. In this case, we consider changes in the penalty from the original 10 to 1, 5, and 20. Again predictably, higher values of λ lead to more conservative strategies. Still, as in the previous case, we observe that none of the strategies selected consist of allocations with less than 40% in stocks.

We believe that our choices for the base case scenario are sensible, as is the fairly broad range of values we consider in our sensitivity analysis. Still, we performed a broader sensitivity analysis with coefficients of risk aversion and penalty coefficients much higher than what would be considered plausible, to explore whether we observed substantial changes in the strategies selected. To illustrate our broader analysis, Exhibit 5 displays the results for the U.S. market.

Exhibit 5: Utility-Based Approach – Broader Sensitivity Analysis – USA

This exhibit shows the proportion of stocks in the asset allocation (AA) selected, the rest of the portfolio invested in bonds, for different values of the risk aversion (γ) and penalty (λ) coefficients. The utility function is given by expression (3). The asset allocations considered range between 100 (all stocks) and 0 (no stocks), with nine allocations (90, 80, …, 20, 10) in between. All strategies are evaluated over 86 rolling 30-year retirement periods between 1900-1929 and 1985-2014; a starting capital of $1,000; a 4% initial withdrawal rate; subsequent annual withdrawals adjusted by inflation; and annual rebalancing. The data is described in Exhibit A1 in the appendix.

γ	**0.5**	**0.9999**	**1.5**	**2.0**	**2.5**	**3.0**	**3.5**	**4.0**	**4.5**	**5.0**	**10.0**	**15.0**	**20.0**
AA	100	100	100	90	90	80	80	80	80	80	80	80	80
λ	**1**	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**	**75**	**100**
AA	100	100	100	100	100	100	90	90	90	90	90	80	80

As Exhibit 5 shows, the asset allocation selected does become more conservative as either γ or λ increase. However, even for very high (and perhaps rather implausible) values of these coefficients, the strategy selected in the U.S. market never allocates less than 80% to stocks.

One final issue we address is the impact of the penalty in the kinked utility function we propose. In particular, we explore the optimal choices that follow from the expression

$$U(C) = \frac{C^{1-\gamma} - 1}{1-\gamma} \qquad \text{for all } C \qquad (4)$$

that is, a power utility function over the entire range of coverage ratios, regardless of whether this variable is above or below 1. This utility function indicates that a retiree's utility decreases as the coverage ratio decreases, but there is no additional penalty when a strategy fails (that is, when C<1). The results of our analysis are shown in Exhibit 6.

A comparison between exhibits 4 and 6 reveals that the strategies selected by a retiree with the utility function given by (4) would be somewhat more aggressive than those selected by another retiree with a utility function given by (3). In other words, a more severe penalty for failure results in allocations with a higher proportion of stocks. On average across all markets, the strategies selected based on (3) allocate 91% to stocks; those selected based on (4), on the other hand, allocate 98% to stocks.

Exhibit 6: Utility-Based Approach – A Modified Utility Function

This exhibit shows the average utility for the 11 asset allocations considered, ranging from 100 (all stocks, no bonds) to 0 (no stocks, all bonds), with nine allocations (90, 80, ..., 20, 10) in between, the rest being allocated to bonds. The utility function is given by expression (4) in the text, for γ=0.9999. All strategies are evaluated over 86 rolling 30-year retirement periods between 1900-1929 and 1985-2014; a starting capital of $1,000; a 4% initial withdrawal rate; subsequent annual withdrawals adjusted by inflation; and annual rebalancing. The data is described in Exhibit A1 in the appendix.

Country	100	90	80	70	60	50	40	30	20	10	0
Australia	1.37	1.25	1.12	0.98	0.83	0.68	0.52	0.35	0.20	0.05	−0.07
Austria	−0.05	0.05	0.10	0.10	0.08	0.04	−0.03	−0.10	−0.19	−0.29	−0.40
Belgium	0.18	0.16	0.15	0.13	0.10	0.07	0.03	−0.02	−0.07	−0.12	−0.18
Canada	1.05	0.98	0.90	0.81	0.70	0.59	0.48	0.36	0.25	0.14	0.04
Denmark	0.75	0.74	0.71	0.68	0.64	0.60	0.54	0.48	0.42	0.35	0.28
Finland	0.70	0.63	0.56	0.47	0.38	0.27	0.16	0.04	−0.08	−0.20	−0.32
France	0.17	0.16	0.15	0.14	0.12	0.10	0.08	0.05	0.02	−0.02	−0.07
Germany	0.17	0.14	0.11	0.08	0.04	0.00	−0.04	−0.10	−0.16	−0.23	−0.31
Ireland	0.63	0.58	0.53	0.46	0.39	0.30	0.22	0.13	0.04	−0.03	−0.09
Italy	−0.11	−0.14	−0.17	−0.20	−0.22	−0.25	−0.28	−0.31	−0.35	−0.40	−0.45
Japan	0.60	0.58	0.55	0.51	0.45	0.38	0.30	0.20	0.10	−0.02	−0.14
Netherlands	0.77	0.73	0.67	0.61	0.53	0.44	0.35	0.26	0.17	0.07	−0.02
New Zealand	1.03	0.96	0.88	0.78	0.68	0.56	0.44	0.32	0.21	0.10	−0.01
Norway	0.38	0.39	0.38	0.36	0.33	0.29	0.23	0.17	0.09	0.01	−0.08
Portugal	0.35	0.34	0.33	0.30	0.28	0.25	0.21	0.15	0.09	0.02	−0.07
South Africa	1.41	1.27	1.13	0.97	0.81	0.65	0.48	0.32	0.16	0.02	−0.10
Spain	0.14	0.15	0.14	0.12	0.11	0.08	0.06	0.02	−0.02	−0.07	−0.12
Sweden	0.97	0.92	0.85	0.77	0.69	0.60	0.50	0.40	0.30	0.21	0.12
Switzerland	0.54	0.52	0.50	0.47	0.42	0.37	0.32	0.26	0.19	0.12	0.05
UK	0.97	0.88	0.77	0.67	0.56	0.45	0.33	0.22	0.12	0.03	−0.05
USA	1.11	1.03	0.95	0.85	0.73	0.61	0.49	0.36	0.23	0.11	0.00
World	0.86	0.78	0.70	0.60	0.51	0.41	0.32	0.23	0.15	0.07	−0.02

To conclude, note that in our analysis we consider the asset allocation between only two asset classes, stocks and bonds. For only two asset classes, it is straightforward to calculate outcomes for all possible combinations and determine the optimal portfolio. Individuals, and particularly advisors, that want to consider more granular allocations must resort to full-scale optimization in order to identify the utility-maximizing allocation. This methodology applies a numerical algorithm based on evolutionary biology to identify the optimal mix of asset classes; see Cremers et al (2005) for a description and illustration of this approach.

4. Assessment

One of the most important decisions investors planning for retirement or already in retirement face is the allocation to risky and safe assets. Academics and practitioners have responded to this challenge by proposing many different approaches to determine an optimal choice. We contribute to this literature by proposing both a variable, the coverage ratio, and an approach, based on utility. We believe our methodology overcomes some of the weaknesses of existing approaches, and may provide advisors with a valuable tool they can use to help clients deal with this critical issue.

We illustrate our utility-based approach by focusing on the retirement period, and particularly on the choice of an optimal asset allocation. However, our approach is general and could be also applied to select an optimal withdrawal rate, as well as a saving rate and asset allocation during the accumulation period.

The variable we introduce, the coverage ratio, aims to summarize both the frequency and the magnitude of the success or failure of a given strategy, thus providing a more comprehensive assessment than that provided by the ubiquitous failure rate. Furthermore, the approach we introduce, based on expected utility, aims to account for the specific preferences of the retiree. We believe that the combination of the coverage ratio and the specific utility function we propose should help advisors provide better recommendations to their clients than they would be able to provide with other existing approaches.

Our empirical results show that retirees in most countries would be well served by selecting fairly aggressive strategies at the beginning of retirement. That may sound scary to many individuals, but over long holding periods the higher compounding power of stocks relative to that of bonds tends to overwhelm other considerations, including a relatively high degree of risk aversion. That said, nothing in our analysis suggests that retirees should not revisit their asset allocation over time, as their holding period shortens.

In fact, we strongly suspect that the generally aggressive strategies we find to be optimal in most countries are closely related to the length of the retirement period we consider, which is rather standard in the literature. Over long periods such as 30 years, stocks are very likely to outperform bonds by a substantial margin, which pushes the choice toward aggressive strategies. That said, it may in fact be wise for a retiree to revisit his asset allocation periodically, which would imply doing it on the basis of an ever shorter retirement period.

Would a retiree reconsidering his asset allocation every five years always lean toward the rather aggressive strategies we find for a 30-year retirement period? We doubt it. Would a retiree's degree of risk aversion have the same impact on the optimal choice when he expects to live ten or five more years than when he expects to live 30 more years? We doubt it. These are important questions somewhat beyond the scope of this article, but we address them in separate research already in progress.

Appendix

Exhibit A1: Summary Statistics

This exhibit shows, for the series of annual returns over the 1900-2014 period, the arithmetic (AM) and geometric (GM) mean return, standard deviation (SD), semideviation for a 0% benchmark (SSD), lowest return (Min), and highest return (Max). All returns are real (adjusted by each country's inflation rate), in local currency (except for the world market, in dollars), and account for capital gains/losses and cash flows (dividends or coupons). All figures in %.

	AM	GM	SD	SSD	Min	Max
A: Stocks						
Australia	8.9	7.3	17.9	9.2	−42.5	51.5
Austria	4.6	0.6	30.0	15.6	−60.1	127.1
Belgium	5.4	2.7	23.7	13.0	−48.9	105.1
Canada	7.2	5.8	16.9	8.4	−33.8	55.2
Denmark	7.2	5.3	20.7	8.9	−49.2	107.8
Finland	9.3	5.3	30.0	13.9	−60.8	161.7
France	5.7	3.2	23.1	12.3	−41.5	66.1
Germany	8.2	3.2	31.7	14.7	−90.8	154.6
Ireland	6.8	4.2	22.9	11.9	−65.4	68.4
Italy	5.9	1.9	28.5	15.6	−72.9	120.7
Japan	8.8	4.1	29.6	15.2	−85.5	121.1
Netherlands	7.1	5.0	21.4	10.3	−50.4	101.6
New Zealand	7.8	6.1	19.4	9.0	−54.7	105.3
Norway	7.2	4.2	26.9	11.7	−53.6	166.9
Portugal	8.4	3.4	34.4	15.3	−76.6	151.8
South Africa	9.5	7.4	22.1	9.0	−52.2	102.9
Spain	5.9	3.7	21.9	11.0	−43.3	99.4
Sweden	8.0	5.8	21.2	10.8	−42.5	67.5
Switzerland	6.3	4.5	19.5	10.1	−37.8	59.4
UK	7.1	5.3	19.6	9.7	−57.1	96.7
USA	8.5	6.5	20.0	10.4	−37.6	56.3
World	6.6	5.2	17.4	9.4	−41.0	68.2
B: Bonds						
Australia	2.5	1.7	13.2	7.6	−26.6	62.2
Austria	4.9	−3.8	51.2	20.1	−94.4	441.6
Belgium	1.6	0.4	15.0	9.9	−45.6	62.3
Canada	2.8	2.2	10.4	5.4	−25.9	41.7
Denmark	3.9	3.3	11.9	5.1	−18.2	50.1
Finland	1.5	0.2	13.7	10.9	−69.5	30.2
France	1.1	0.2	13.0	9.5	−43.5	35.9
Germany	1.3	−1.4	15.8	12.4	−95.0	62.5
Ireland	2.7	1.6	15.1	8.0	−34.1	61.2
Italy	0.2	−1.2	14.7	11.8	−64.3	35.5
Japan	1.7	−0.9	19.7	14.7	−77.5	69.8
Netherlands	2.2	1.7	9.8	5.2	−18.1	32.8
New Zealand	2.5	2.1	9.0	4.8	−23.7	34.1
Norway	2.6	1.9	12.0	6.8	−48.0	62.1
Portugal	2.5	0.8	18.7	11.2	−49.7	82.4
South Africa	2.4	1.9	10.4	5.9	−32.6	37.1
Spain	2.5	1.8	12.6	7.1	−30.2	53.2
Sweden	3.5	2.8	12.7	5.9	−37.0	68.2
Switzerland	2.7	2.3	9.4	4.3	−21.4	56.1
UK	2.4	1.6	13.7	7.1	−30.7	59.4
USA	2.5	2.0	10.4	5.3	−18.4	35.1
World	2.5	1.9	11.3	6.0	−32.0	46.7

Exhibit A2: Coverage Ratio – Utility-Based Approach – Sensitivity Analysis on γ

This exhibit shows the average utility for the 11 asset allocations considered, ranging from 100 (all stocks) to 0 (no stocks), with nine allocations (90, 80, ..., 20, 10) in between, the rest being allocated to bonds. The utility function is given by expression (3) in the text, for γ=0 and λ=10. All strategies are evaluated over 86 rolling 30-year retirement periods between 1900-1929 and 1985-2014; a starting capital of $1,000; a 4% initial withdrawal rate; subsequent annual withdrawals adjusted by inflation; and annual rebalancing. The data is described in Exhibit A1.

Country	100	90	80	70	60	50	40	30	20	10	0
Australia	3.94	3.30	2.72	2.18	1.66	1.14	0.53	−0.09	−0.70	−1.41	−1.96
Austria	−1.91	−1.53	−1.35	−1.37	−1.47	−1.63	−1.83	−2.08	−2.35	−2.66	−2.99
Belgium	−0.68	−0.73	−0.79	−0.89	−1.04	−1.19	−1.36	−1.54	−1.75	−1.97	−2.23
Canada	2.31	2.04	1.76	1.51	1.28	0.96	0.60	0.25	−0.19	−0.81	−1.34
Denmark	1.54	1.57	1.56	1.53	1.47	1.35	1.16	0.92	0.56	0.17	−0.26
Finland	1.69	1.28	0.87	0.39	−0.06	−0.50	−0.95	−1.40	−1.80	−2.21	−2.69
France	−0.55	−0.69	−0.82	−0.91	−1.02	−1.12	−1.26	−1.42	−1.57	−1.79	−2.00
Germany	−0.62	−0.86	−1.13	−1.35	−1.56	−1.77	−1.95	−2.16	−2.35	−2.56	−2.76
Ireland	1.08	0.89	0.68	0.46	0.22	−0.09	−0.48	−0.92	−1.40	−1.81	−2.16
Italy	−2.05	−2.16	−2.26	−2.41	−2.51	−2.64	−2.76	−2.90	−3.04	−3.24	−3.45
Japan	1.85	1.35	0.88	0.45	0.08	−0.25	−0.58	−0.84	−1.09	−1.36	−1.80
Netherlands	1.72	1.56	1.37	1.17	0.91	0.59	0.21	−0.24	−0.71	−1.18	−1.61
New Zealand	2.14	1.90	1.67	1.43	1.19	0.90	0.55	0.10	−0.36	−0.83	−1.37
Norway	0.31	0.36	0.30	0.18	0.00	−0.22	−0.49	−0.76	−1.13	−1.53	−1.93
Portugal	−0.26	−0.21	−0.14	−0.14	−0.17	−0.36	−0.66	−1.04	−1.40	−1.74	−2.09
South Africa	3.89	3.18	2.55	2.01	1.51	1.07	0.63	0.07	−0.47	−1.09	−1.74
Spain	−0.45	−0.43	−0.48	−0.53	−0.62	−0.75	−0.91	−1.11	−1.34	−1.62	−1.88
Sweden	2.80	2.52	2.22	1.92	1.64	1.34	1.07	0.70	0.21	−0.33	−0.87
Switzerland	0.30	0.28	0.26	0.20	0.12	0.04	−0.04	−0.15	−0.28	−0.44	−0.69
UK	2.16	1.81	1.43	1.05	0.70	0.37	0.01	−0.46	−1.03	−1.56	−1.99
USA	2.63	2.29	1.96	1.64	1.34	1.05	0.74	0.38	−0.12	−0.75	−1.36
World	1.57	1.30	1.02	0.71	0.41	0.10	−0.23	−0.57	−0.85	−1.10	−1.49

Exhibit A3: Coverage Ratio – Utility-Based Approach – Sensitivity Analysis on γ

This exhibit shows the average utility for the 11 asset allocations considered, ranging from 100 (all stocks) to 0 (no stocks), with nine allocations (90, 80, ..., 20, 10) in between, the rest being allocated to bonds. The utility function is given by expression (3) in the text, for γ=0.5 and λ=10. All strategies are evaluated over 86 rolling 30-year retirement periods between 1900-1929 and 1985-2014; a starting capital of $1,000; a 4% initial withdrawal rate; subsequent annual withdrawals adjusted by inflation; and annual rebalancing. The data is described in Exhibit A1.

Country	100	90	80	70	60	50	40	30	20	10	0
Australia	2.12	1.86	1.59	1.31	1.00	0.64	0.16	−0.35	−0.90	−1.55	−2.06
Austria	−2.11	−1.78	−1.62	−1.65	−1.75	−1.90	−2.06	−2.27	−2.50	−2.76	−3.06
Belgium	−1.09	−1.12	−1.16	−1.23	−1.35	−1.46	−1.60	−1.74	−1.91	−2.10	−2.33
Canada	1.50	1.37	1.23	1.08	0.92	0.66	0.35	0.04	−0.38	−0.98	−1.50
Denmark	1.01	1.02	0.99	0.96	0.91	0.81	0.65	0.43	0.12	−0.23	−0.60
Finland	0.11	−0.12	−0.34	−0.61	−0.86	−1.11	−1.39	−1.69	−1.98	−2.32	−2.74
France	−0.99	−1.16	−1.32	−1.42	−1.52	−1.61	−1.72	−1.83	−1.93	−2.10	−2.26
Germany	−1.54	−1.64	−1.78	−1.89	−2.00	−2.11	−2.22	−2.36	−2.49	−2.65	−2.82
Ireland	0.33	0.20	0.06	−0.09	−0.27	−0.51	−0.84	−1.22	−1.65	−2.01	−2.31
Italy	−2.16	−2.26	−2.37	−2.52	−2.61	−2.75	−2.86	−2.99	−3.11	−3.30	−3.50
Japan	−0.08	−0.17	−0.28	−0.40	−0.54	−0.68	−0.87	−1.04	−1.23	−1.45	−1.87
Netherlands	0.81	0.77	0.70	0.61	0.45	0.22	−0.08	−0.46	−0.87	−1.30	−1.68
New Zealand	1.45	1.32	1.18	1.03	0.86	0.63	0.33	−0.07	−0.49	−0.92	−1.45
Norway	−0.24	−0.20	−0.23	−0.32	−0.45	−0.60	−0.82	−1.01	−1.33	−1.68	−2.04
Portugal	−0.90	−0.76	−0.62	−0.55	−0.52	−0.66	−0.90	−1.24	−1.55	−1.86	−2.19
South Africa	2.22	1.92	1.62	1.33	1.05	0.76	0.43	−0.06	−0.55	−1.14	−1.77
Spain	−0.96	−0.91	−0.91	−0.91	−0.94	−1.01	−1.13	−1.28	−1.46	−1.71	−1.94
Sweden	1.01	0.96	0.89	0.82	0.75	0.64	0.54	0.30	−0.08	−0.53	−1.02
Switzerland	−0.10	−0.06	−0.03	−0.04	−0.08	−0.12	−0.16	−0.24	−0.34	−0.49	−0.73
UK	1.35	1.14	0.88	0.59	0.32	0.06	−0.25	−0.68	−1.21	−1.71	−2.12
USA	1.60	1.47	1.31	1.13	0.94	0.74	0.50	0.18	−0.27	−0.87	−1.46
World	0.98	0.81	0.61	0.37	0.13	−0.15	−0.45	−0.75	−1.02	−1.24	−1.61

Exhibit A4: Coverage Ratio – Utility-Based Approach – Sensitivity Analysis on γ

This exhibit shows the average utility for the 11 asset allocations considered, ranging from 100 (all stocks) to 0 (no stocks), with nine allocations (90, 80, …, 20, 10) in between, the rest being allocated to bonds. The utility function is given by expression (3) in the text, for γ=2 and λ=10. All strategies are evaluated over 86 rolling 30-year retirement periods between 1900-1929 and 1985-2014; a starting capital of $1,000; a 4% initial withdrawal rate; subsequent annual withdrawals adjusted by inflation; and annual rebalancing. The data is described in Exhibit A1.

Country	100	90	80	70	60	50	40	30	20	10	0
Australia	0.54	0.51	0.45	0.37	0.24	0.04	−0.30	−0.70	−1.16	−1.75	−2.21
Austria	−2.38	−2.08	−1.96	−2.01	−2.10	−2.23	−2.36	−2.53	−2.71	−2.92	−3.18
Belgium	−1.47	−1.50	−1.52	−1.57	−1.67	−1.76	−1.86	−1.97	−2.11	−2.27	−2.46
Canada	0.58	0.58	0.54	0.50	0.44	0.26	0.02	−0.25	−0.63	−1.20	−1.69
Denmark	0.42	0.44	0.42	0.39	0.36	0.28	0.14	−0.04	−0.33	−0.64	−0.98
Finland	−1.02	−1.16	−1.27	−1.42	−1.54	−1.66	−1.82	−2.01	−2.20	−2.45	−2.81
France	−1.39	−1.59	−1.76	−1.87	−1.98	−2.06	−2.15	−2.24	−2.31	−2.44	−2.56
Germany	−2.17	−2.22	−2.32	−2.38	−2.44	−2.51	−2.55	−2.64	−2.71	−2.81	−2.93
Ireland	−0.36	−0.43	−0.52	−0.62	−0.73	−0.92	−1.19	−1.54	−1.92	−2.25	−2.51
Italy	−2.32	−2.41	−2.51	−2.66	−2.75	−2.88	−2.98	−3.10	−3.21	−3.39	−3.57
Japan	−1.26	−1.23	−1.21	−1.20	−1.20	−1.21	−1.29	−1.35	−1.44	−1.61	−1.99
Netherlands	−0.03	0.03	0.04	0.05	−0.03	−0.18	−0.41	−0.73	−1.08	−1.46	−1.80
New Zealand	0.60	0.58	0.54	0.48	0.41	0.26	0.03	−0.31	−0.68	−1.07	−1.56
Norway	−0.74	−0.70	−0.73	−0.79	−0.89	−1.00	−1.16	−1.31	−1.57	−1.88	−2.20
Portugal	−1.51	−1.30	−1.09	−0.97	−0.90	−0.99	−1.20	−1.50	−1.78	−2.06	−2.36
South Africa	0.68	0.64	0.59	0.53	0.44	0.32	0.12	−0.26	−0.68	−1.21	−1.82
Spain	−1.34	−1.26	−1.25	−1.21	−1.22	−1.26	−1.34	−1.46	−1.61	−1.82	−2.03
Sweden	−0.30	−0.20	−0.13	−0.05	0.01	0.04	0.05	−0.09	−0.39	−0.77	−1.21
Switzerland	−0.61	−0.53	−0.45	−0.40	−0.38	−0.36	−0.36	−0.40	−0.46	−0.58	−0.80
UK	0.47	0.38	0.23	0.05	−0.13	−0.32	−0.57	−0.96	−1.45	−1.92	−2.29
USA	0.54	0.54	0.53	0.48	0.42	0.32	0.17	−0.07	−0.47	−1.04	−1.59
World	0.24	0.17	0.07	−0.10	−0.27	−0.49	−0.74	−1.00	−1.23	−1.43	−1.77

Exhibit A5: Coverage Ratio – Utility-Based Approach – Sensitivity Analysis on λ

This exhibit shows the average utility for the 11 asset allocations considered, ranging from 100 (all stocks) to 0 (no stocks), with nine allocations (90, 80, …, 20, 10) in between, the rest being allocated to bonds. The utility function is given by expression (3) in the text, for γ=0.9999 and λ=1. All strategies are evaluated over 86 rolling 30-year retirement periods between 1900-1929 and 1985-2014; a starting capital of $1,000; a 4% initial withdrawal rate; subsequent annual withdrawals adjusted by inflation; and annual rebalancing. The data is described in Exhibit A1.

Country	100	90	80	70	60	50	40	30	20	10	0
Australia	1.38	1.26	1.12	0.98	0.84	0.68	0.53	0.38	0.23	0.10	0.00
Austria	0.10	0.18	0.22	0.24	0.23	0.20	0.15	0.10	0.04	−0.03	−0.11
Belgium	0.24	0.23	0.22	0.20	0.17	0.14	0.11	0.07	0.03	−0.02	−0.06
Canada	1.05	0.98	0.90	0.81	0.70	0.59	0.48	0.37	0.26	0.16	0.08
Denmark	0.75	0.74	0.71	0.68	0.64	0.60	0.54	0.49	0.42	0.36	0.30
Finland	0.77	0.71	0.64	0.56	0.47	0.38	0.27	0.16	0.05	−0.06	−0.16
France	0.23	0.23	0.22	0.22	0.21	0.20	0.18	0.16	0.14	0.11	0.08
Germany	0.31	0.29	0.26	0.24	0.21	0.18	0.14	0.10	0.05	−0.01	−0.08
Ireland	0.65	0.60	0.55	0.48	0.41	0.33	0.25	0.17	0.10	0.04	−0.01
Italy	0.00	−0.03	−0.05	−0.08	−0.09	−0.12	−0.14	−0.16	−0.19	−0.23	−0.26
Japan	0.73	0.71	0.68	0.63	0.57	0.50	0.42	0.33	0.23	0.12	0.00
Netherlands	0.78	0.73	0.68	0.61	0.53	0.45	0.36	0.27	0.19	0.10	0.03
New Zealand	1.03	0.96	0.88	0.78	0.68	0.56	0.45	0.33	0.23	0.12	0.03
Norway	0.41	0.42	0.41	0.39	0.36	0.32	0.26	0.20	0.13	0.06	−0.01
Portugal	0.43	0.41	0.38	0.35	0.32	0.30	0.26	0.22	0.17	0.11	0.04
South Africa	1.41	1.27	1.13	0.97	0.81	0.65	0.48	0.32	0.18	0.05	−0.06
Spain	0.21	0.21	0.19	0.18	0.15	0.13	0.10	0.06	0.02	−0.03	−0.07
Sweden	1.01	0.94	0.87	0.78	0.70	0.60	0.51	0.41	0.31	0.22	0.14
Switzerland	0.57	0.55	0.52	0.48	0.44	0.39	0.33	0.27	0.21	0.14	0.06
UK	0.97	0.88	0.78	0.67	0.57	0.46	0.35	0.25	0.16	0.08	0.02
USA	1.11	1.03	0.95	0.85	0.73	0.61	0.49	0.36	0.24	0.13	0.03
World	0.86	0.79	0.70	0.61	0.52	0.43	0.35	0.27	0.19	0.12	0.04

Exhibit A6: Coverage Ratio – Utility-Based Approach – Sensitivity Analysis on λ

This exhibit shows the average utility for the 11 asset allocations considered, ranging from 100 (all stocks) to 0 (no stocks), with nine allocations (90, 80, …, 20, 10) in between, the rest being allocated to bonds. The utility function is given by expression (3) in the text, for γ=0.9999 and λ=5. All strategies are evaluated over 86 rolling 30-year retirement periods between 1900-1929 and 1985-2014; a starting capital of $1,000; a 4% initial withdrawal rate; subsequent annual withdrawals adjusted by inflation; and annual rebalancing. The data is described in Exhibit A1.

Country	100	90	80	70	60	50	40	30	20	10	0
Australia	1.32	1.20	1.07	0.91	0.74	0.53	0.26	−0.02	−0.33	−0.67	−0.94
Austria	−0.94	−0.76	−0.67	−0.68	−0.73	−0.80	−0.89	−1.01	−1.13	−1.28	−1.45
Belgium	−0.44	−0.46	−0.48	−0.52	−0.58	−0.64	−0.71	−0.79	−0.88	−0.98	−1.10
Canada	1.04	0.98	0.90	0.81	0.70	0.54	0.35	0.16	−0.08	−0.39	−0.66
Denmark	0.74	0.73	0.71	0.67	0.63	0.57	0.47	0.35	0.18	0.00	−0.19
Finland	0.18	0.08	−0.03	−0.16	−0.28	−0.41	−0.57	−0.73	−0.90	−1.09	−1.32
France	−0.41	−0.50	−0.57	−0.62	−0.67	−0.71	−0.77	−0.82	−0.87	−0.95	−1.03
Germany	−0.67	−0.71	−0.77	−0.82	−0.87	−0.93	−0.98	−1.05	−1.12	−1.22	−1.32
Ireland	0.34	0.27	0.19	0.10	0.00	−0.14	−0.32	−0.52	−0.74	−0.92	−1.08
Italy	−0.99	−1.05	−1.11	−1.19	−1.24	−1.31	−1.37	−1.44	−1.51	−1.61	−1.72
Japan	0.06	0.05	0.02	−0.02	−0.07	−0.14	−0.24	−0.34	−0.46	−0.61	−0.86
Netherlands	0.59	0.58	0.53	0.48	0.39	0.26	0.09	−0.11	−0.33	−0.55	−0.76
New Zealand	1.03	0.96	0.87	0.78	0.66	0.52	0.33	0.10	−0.13	−0.37	−0.65
Norway	0.00	0.02	0.01	−0.04	−0.11	−0.18	−0.30	−0.40	−0.57	−0.75	−0.95
Portugal	−0.30	−0.24	−0.17	−0.15	−0.14	−0.20	−0.32	−0.49	−0.65	−0.81	−0.98
South Africa	1.40	1.26	1.11	0.96	0.79	0.61	0.40	0.12	−0.17	−0.49	−0.83
Spain	−0.41	−0.38	−0.38	−0.38	−0.40	−0.44	−0.50	−0.57	−0.67	−0.80	−0.92
Sweden	0.67	0.65	0.62	0.58	0.54	0.47	0.40	0.27	0.07	−0.16	−0.41
Switzerland	0.16	0.18	0.19	0.18	0.15	0.12	0.07	0.01	−0.06	−0.16	−0.30
UK	0.94	0.82	0.68	0.52	0.36	0.20	0.01	−0.23	−0.50	−0.76	−0.97
USA	1.08	1.02	0.94	0.83	0.72	0.58	0.42	0.23	−0.03	−0.35	−0.66
World	0.75	0.66	0.55	0.41	0.26	0.10	−0.07	−0.24	−0.39	−0.53	−0.72

Exhibit A7: Coverage Ratio – Utility-Based Approach – Sensitivity Analysis on λ

This exhibit shows the average utility for the 11 asset allocations considered, ranging from 100 (all stocks) to 0 (no stocks), with nine allocations (90, 80, …, 20, 10) in between, the rest being allocated to bonds. The utility function is given by expression (3) in the text, for γ=0.9999 and λ=20. All strategies are evaluated over 86 rolling 30-year retirement periods between 1900-1929 and 1985-2014; a starting capital of $1,000; a 4% initial withdrawal rate; subsequent annual withdrawals adjusted by inflation; and annual rebalancing. The data is described in Exhibit A1.

Country	100	90	80	70	60	50	40	30	20	10	0
Australia	1.11	0.99	0.84	0.65	0.37	−0.03	−0.72	−1.52	−2.41	−3.58	−4.49
Austria	−4.84	−4.26	−4.02	−4.12	−4.31	−4.56	−4.82	−5.17	−5.52	−5.95	−6.46
Belgium	−3.00	−3.05	−3.09	−3.20	−3.39	−3.57	−3.78	−4.00	−4.27	−4.58	−4.98
Canada	1.02	0.98	0.90	0.81	0.69	0.35	−0.12	−0.62	−1.35	−2.48	−3.45
Denmark	0.67	0.70	0.68	0.64	0.59	0.46	0.20	−0.15	−0.72	−1.33	−2.01
Finland	−2.00	−2.29	−2.53	−2.86	−3.12	−3.39	−3.72	−4.10	−4.47	−4.96	−5.68
France	−2.81	−3.20	−3.54	−3.76	−3.97	−4.14	−4.33	−4.51	−4.66	−4.93	−5.18
Germany	−4.36	−4.47	−4.67	−4.80	−4.94	−5.08	−5.18	−5.37	−5.52	−5.74	−5.97
Ireland	−0.80	−0.95	−1.13	−1.32	−1.54	−1.91	−2.44	−3.11	−3.89	−4.55	−5.08
Italy	−4.72	−4.89	−5.08	−5.37	−5.54	−5.80	−6.01	−6.25	−6.46	−6.80	−7.17
Japan	−2.47	−2.44	−2.43	−2.45	−2.48	−2.54	−2.72	−2.85	−3.04	−3.34	−4.07
Netherlands	−0.11	−0.02	−0.01	−0.01	−0.16	−0.46	−0.92	−1.55	−2.25	−3.01	−3.69
New Zealand	1.03	0.96	0.87	0.76	0.62	0.34	−0.09	−0.75	−1.47	−2.23	−3.20
Norway	−1.54	−1.46	−1.50	−1.64	−1.84	−2.06	−2.39	−2.68	−3.20	−3.82	−4.46
Portugal	−3.06	−2.66	−2.25	−2.02	−1.87	−2.07	−2.50	−3.11	−3.69	−4.24	−4.83
South Africa	1.36	1.22	1.06	0.90	0.70	0.45	0.07	−0.66	−1.47	−2.52	−3.70
Spain	−2.72	−2.58	−2.54	−2.48	−2.49	−2.57	−2.72	−2.96	−3.27	−3.69	−4.11
Sweden	−0.57	−0.42	−0.30	−0.17	−0.06	−0.01	0.03	−0.25	−0.84	−1.62	−2.49
Switzerland	−1.36	−1.20	−1.05	−0.97	−0.94	−0.91	−0.90	−0.95	−1.06	−1.26	−1.67
UK	0.82	0.62	0.31	−0.06	−0.42	−0.77	−1.24	−1.99	−2.97	−3.91	−4.66
USA	0.98	0.96	0.90	0.79	0.65	0.47	0.18	−0.27	−1.04	−2.15	−3.24
World	0.35	0.19	−0.03	−0.37	−0.71	−1.13	−1.62	−2.13	−2.57	−2.94	−3.60

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